

INTEGRABILITY OF THE MAJORANT OF PARTIAL SUMS OF A TRIGONOMETRIC SERIES WITH QUASICONVEX COEFFICIENTS

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Abstract

Full Text

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MATHEMATICS

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INTEGRABILITY OF THE MAJORANT OF PARTIAL SUMS OF A TRIGONOMETRIC SERIES WITH QUASICONVEX COEFFICIENTS

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Let the numbers a_k , $k = 0, 1, 2, \dots$, tend to zero and form a quasiconvex sequence, i.e. the series converges

$$\sum_{k=1}^{\infty} k |\Delta^2 a_{k-1}|, \quad \text{where } \Delta^2 a_{k-1} = a_{k-1} - 2a_k + a_{k+1}.$$

Then the series

$$\frac{a_0}{2} + \sum_{k=1}^{\infty} a_k \cos kx, \quad \sum_{k=1}^{\infty} a_k \sin kx \quad (1)$$

converge for all $x \in (0, \pi]$ to functions continuous for these x , which we shall denote respectively by $f(x)$ and $g(x)$.

It is known that under the assumptions made $f \in L[0, \pi]$ (⁽¹⁾; (⁽²⁾, § 5.12), and for the integral of the modulus of the function g the estimate (⁽³⁾) is valid

$$\int_{\varepsilon}^{\pi} |g(x)| dx = \sum_{k=1}^{[1/\varepsilon]} \frac{|a_k|}{k} + O(1), \quad (2)$$

whence it follows that $g \in L[0, \pi]$ if and only if the series

$$\sum_{k=1}^{\infty} \frac{|a_k|}{k}$$

converges.

Consider the question of integrability of the majorants of the partial sums of the series (1), i.e. of the functions

$$f^*(x) = \max_n \left| \frac{a_0}{2} + \sum_{k=1}^n a_k \cos kx \right|, \quad x \neq 0,$$

$$g^*(x) = \max_n \left| \sum_{k=1}^n a_k \sin kx \right|.$$

Theorem. Let $\{a_k\}$ be a quasiconvex sequence of numbers tending to zero. Then, as $\varepsilon \rightarrow +0$, the estimates

$$\int_{\varepsilon}^{\pi} f^*(x) dx = \sum_{k=1}^{\lfloor 1/\varepsilon \rfloor} \frac{1}{k} \max_{n \geq k} |a_n| + O(1), \quad (3)$$

$$\int_{\varepsilon}^{\pi} g^*(x) dx = \sum_{k=1}^{\lfloor 1/\varepsilon \rfloor} \frac{1}{k} \left(|a_k| + \max_{n \geq k} |a_n| \right) + O(1). \quad (4)$$

In particular, each of the functions f^* and g^* belongs to $L[0, \pi]$ if and only if the series

$$\sum_{k=1}^{\infty} \frac{1}{k} \max_{n \geq k} |a_n| \quad (5)$$

converges.

From estimates (2) and (4) we conclude that, for series with quasiconvex coefficients,

$$\int_{\varepsilon}^{\pi} g^*(x) dx \geq 2 \int_{\varepsilon}^{\pi} |g(x)| dx + O(1), \quad (6)$$

and if one additionally assumes that a_k decrease monotonically, then

$$\int_{\varepsilon}^{\pi} g^*(x) dx = 2 \int_{\varepsilon}^{\pi} |g(x)| dx + O(1). \quad (7)$$

Let us note that from the known results for series with monotone coefficients it follows that relation (7) holds even without the assumption of quasiconvexity of $\{a_k\}$, under the sole condition that a_k decrease monotonically.

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References

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Note: Figure translations are in progress. See original paper for figures.

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