

ON THE LIMITING DEFLECTION OF A VISCOELASTIC BEAM

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Abstract

Full Text

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THEORY OF ELASTICITY

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ON THE LIMITING DEFLECTION OF A VISCOELASTIC BEAM

(Presented by Academician Yu. N. Rabotnov on 18 V 1970)

1°. We shall consider a viscoelastic beam of variable cross-section and finite length l , lying on an elastic foundation of Winkler type.

As is known ^(5,8), the deflection $y(x, t)$ of its axis is described by the following boundary-value problem:

$$\begin{aligned} & \frac{\partial^2}{\partial x^2} \left(EI(x) \frac{\partial^2 y}{\partial x^2} \right) + Cy(x, t) + C \int_0^t K(t, \tau) y(x, \tau) d\tau = \\ & = p(x, t) + \int_0^t K(t, \tau) p(x, \tau) d\tau; \quad 0 \leq x \leq l; \quad 0 \leq t < \infty, \end{aligned} \quad (1)$$

$$U_i[y] = 0, \quad (2)$$

where (2) are the boundary conditions describing the nature of the fastening of the beam at its ends, $p(x, t)$ is the external transverse load, $K(t, \tau)$ is the kernel characterizing the hereditary properties of the material of the beam, $I(x)$ is the moment of inertia of the beam cross-section relative to the axis, C is the foundation coefficient, and E is the instantaneous modulus of elasticity. We shall assume that E and C are constants.

Let $\alpha(t)$ be some continuous function, positive for $t \geq 0$. We shall assume that

a)

$$\sup_{0 \leq t < \infty} \int_0^t |K(t, \tau)| \frac{\alpha(t)}{\alpha(\tau)} d\tau < \infty;$$

b)

$$\lim_{t \rightarrow \infty} \int_E K(t, \tau) \frac{\alpha(t)}{\alpha(\tau)} d\tau = 0$$

for any measurable bounded set $E \subset [0, \infty)$;

c)

$$l_K = \lim_{t \rightarrow \infty} \int_0^t K(t, \tau) \frac{\alpha(t)}{\alpha(\tau)} d\tau$$

exists.

Let us note that if $K(t, \tau) = K_0(t - \tau)$ and

$$\alpha(t) = e^{-\theta t}, \quad (3)$$

where θ is some real number, then conditions a), b), c) are equivalent to the following:

$$\int_0^\infty |K_1(t)| e^{-\theta t} dt < \infty. \quad (4)$$

We are interested in the conditions under which the existence of the limit, uniform in x ,

$$(Lp)(x) = \lim_{t \rightarrow \infty} p(x, t) \alpha(t) \quad (5)$$

implies the existence of the limit, uniform in x ,

$$(Ly)(x) = \lim_{t \rightarrow \infty} y(x, t) \alpha(t) \quad (5'')$$

and how to find this latter limit.

In particular, we are interested in the case when the equality

$$(Lp)(x) \equiv 0 \quad (6)$$

implies the equality

$$(Ly)(x) \equiv 0. \quad (6'')$$

2°. Let us denote by $Q(x, \xi)$ the Green's function of the differential operator Λ_x :

$$\Lambda_x[y] = \frac{d^2}{dx^2} \left(I(x) \frac{d^2 y}{dx^2} \right) + \frac{C}{E} y$$

under the boundary conditions (2), and by Q the Fredholm operator acting in the Banach space $C[0, l]$:

$$(Qg)(x) = \int_0^l Q(x, \xi)g(\xi) d\xi. \quad (7)$$

With the aid of Tauberian theorems of the type of Paley–Wiener–Gelfand (^{1–4}), the following propositions can be proved.

Theorem 1. If

$$|E/Cq_0| > \lim_{s \rightarrow \infty} \sup_{s \leq t < \infty} \int_s^t |K(t, \tau)| \frac{\alpha(t)}{\alpha(\tau)} d\tau, \quad (8)$$

where q_0 is the eigenvalue of the operator Q largest in modulus, then the existence of the limit (5) implies the existence of (5''). Moreover,

$$(Ly)(x) = \frac{1}{C} \left(l_{kQ} + \frac{E}{C} I \right)^{-1} Q(Lp + LVp)(x). \quad (9)$$

Condition (8) is also sufficient for (6) to imply (6'').

Theorem 2. Let $\alpha(t)$ have the form (3) and

$$K(t, \tau) = K_0(t - \tau) + \tilde{K}(t, \tau), \quad (10)$$

where $K_0(t)$ satisfies condition (4), and $\tilde{K}(t, \tau)$ satisfies conditions a), b), c) and also the condition

$$\lim_{s \rightarrow \infty} \sup_{s \leq t < \infty} \int_s^t |\tilde{K}(t, \tau)| e^{-\theta(t-\tau)} d\tau = 0. \quad (11)$$

Then the condition

$$-\frac{E}{C} \neq q_i k_0(W); \quad \operatorname{Re} W \geq 0; \quad q_i \in \sigma(Q), \quad (12)$$

where $k_0(W)$ is the Laplace transform of the function $K_0(t)$, and $\sigma(Q)$ is the spectrum of the operator Q (7) in $C[0, l]$, is sufficient for the existence of the limit (5) to imply the existence of the limit (5''). In this case formula (9) holds.

If

$$k_0(W) + 1 \neq 0; \quad \operatorname{Re} W \geq 0, \quad (13)$$

then condition (12) is not only sufficient but also necessary for this.

Theorem 3. Let

- 1) $\alpha(0) = 1$; $\alpha(t + \tau) \leq \alpha(t) \cdot \alpha(\tau)$; $\theta = \lim_{t \rightarrow \infty} \frac{\ln \alpha(t)}{-t} < \infty$;
- 2) $K(t, \tau)$ be representable in the form (10);
- 3) $\tilde{K}(t, \tau)$ satisfy conditions a), b), c) and

$$\lim_{s \rightarrow \infty} \sup_{t < t \leq \infty} \int_s^t |\tilde{K}(t, \tau)| \frac{\alpha(t)}{\alpha(\tau)} d\tau = 0;$$

4)

$$\int_0^\infty |K_0(t)| \alpha(t) dt < \infty.$$

Then condition (12) is sufficient for (6) to imply (6").

Analogous results are obtained when the problem of buckling of a viscoelastic rod is investigated by our method ⁶.

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