

ON THE EXISTENCE OF FUNDAMENTAL DOMAINS OF A FINITELY GENERATED KLEINIAN GROUP

M. A. Saifutdinov

1970

SovietRxiv

View the original and related papers at <https://sovietrxiv.org/items/ru-197001.59039>

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

Abstract

Full Text

UDC 517.5

MATHEMATICS

M. A. SAIFUTDINOV

ON THE EXISTENCE OF FUNDAMENTAL DOMAINS OF A FINITELY GENERATED KLEINIAN GROUP

(Presented by Academician M. A. Lavrent'ev, June 2, 1969)

§ 1. B. Maskit in the paper ⁽¹⁾ formulated the problem of the existence of a fundamental domain of a Kleinian group with a finite number of generators. Below a solution of this problem is given.

We shall give the definitions and results known to us that are needed. By a fundamental domain in a locally compact metric space Y with respect to a discrete group of homeomorphisms G we shall mean an open set $F \subset Y$ satisfying the following two conditions: 1) for any $g_1 \neq g_2$ the sets $g_1 \overline{F}$ and $g_2 \overline{F}$, where $\overline{F}$ is the closure of the set F , have no common elements; 2) the union of the sets gF , where g runs through the group G , coincides with the whole space Y .

A group of Möbius transformations is called discontinuous at a point z if z has a neighborhood U such that the set $g(U) \cap U = \emptyset$ for any $g \in G$. If a group of Möbius transformations is discontinuous at some point $z \in Y$, then it is called a Kleinian group. The set of points Y at which G is discontinuous forms the regular set $R(G)$. The set of points $z \in Y$ for which there exist a point $z_0 \in Y$ and a sequence of distinct elements $\{g_n\}$ from G having the property that $g_n(z) \rightarrow z_0$ as $n \rightarrow \infty$ is called the limit set $L(G)$.

It is known (see ⁽³⁾) that $R(G)$ is an open set, $L(G)$ is a closed set, and $R(G) \cap L(G) = \emptyset$. These sets are invariant with respect to G .

Fundamental domains of Fuchsian groups of the first and second kinds, which are special varieties of Kleinian groups, are described, for example, in ⁽²⁾.

§ 2. Our aim is to prove the following theorem.

Theorem. *A finitely generated Kleinian group has fundamental domains.*

Proof. Consider the quotient space $S = R(G)/G$. It has a natural complex structure such that the projection mapping $\pi : R(G) \rightarrow S$ is holomorphic. Thus, the components S_i of the quotient space S are Riemann surfaces. Let

$R_i = \pi^{-1}S_i$. Generally speaking, R_i are not connected. We shall denote the components of R_i by R_{ij} .

Thus, for any Kleinian group G we have associated decompositions

$$R = \bigcup_i R_i = \bigcup_{i,j} R_{ij}, \quad S = \bigcup S_i.$$

Each R_i is invariant with respect to transformations of the whole group G . The boundaries of R_i lie in $L(G)$. The components R_{ij} are branched covering surfaces over S_i , whose branch points are elliptic fixed points (see (3)).

Each connected component R_{ij} has at least three boundary points. In the case when the number of boundary points is less than three, we have a Fuchsian group, for which a fundamental domain exists. The R_{ij} have universal covering surfaces V_{ij} , which conform-

are mapped onto the disk $|w| < 1$. The Poincaré metric ρ , defined in the disk $|w| < 1$, is transferred to the universal covering surface V_{ij} and, in particular, to R_{ij} (see (4)). The metric ρ , defined on each R_{ij} , gives a metric on all of R . The projection π induces the corresponding metric $\bar{\rho}$ on S . This metric $\bar{\rho}$ has the property of intrinsicness, i.e., for any two points $y_1, y_2 \in Y$ there exists a third point $y_0 \in Y$ such that

$$\bar{\rho}(y_1, y_0) = \bar{\rho}(y_0, y_2) = \frac{1}{2}\bar{\rho}(y_1, y_2).$$

The locally compact metric space Y with such a metric $\bar{\rho}$ has fundamental domains (see (5)). The theorem is proved. This theorem gives a solution to the problem of B. Maskit stated above.

Novosibirsk
State University

Received
31 III 1969

REFERENCES

1. B. Maskit, Trans. Am. Math. Soc., 120, No. 3, 499 (1965).
2. R. Ford, *Automorphic Functions*, 1936.
3. L. Ahlfors, Am. J. Math., 86, No. 2, 413 (1964).
4. S. Stoilow, *The Theory of Functions of a Complex Variable*, 2, IL, 1962.
5. I. M. Gelfand, M. I. Graev, I. I. Pyatetskii-Shapiro, *Representation Theory and Automorphic Functions*, Nauka, 1966.

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.