

ESTIMATING THE CLOSENESS OF FUNCTIONS OF BOUNDED VARIATION FROM THE CLOSENESS OF THEIR FOURIER- STIELTJES TRANSFORMS

V. V. Petrov

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Abstract

Full Text

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MATHEMATICS

V. V. PETROV

ESTIMATING THE CLOSENESS OF FUNCTIONS OF BOUNDED VARIATION FROM THE CLOSENESS OF THEIR FOURIER-STIELTJES TRANSFORMS

(Presented by Academician Yu. V. Linnik on 1 XII 1969)

An important role in probability theory is played by Esseen's inequalities^(1,2), which make it possible to estimate the closeness of a nondecreasing bounded function $F(x)$ and a function of bounded variation $G(x)$ from the closeness of the corresponding Fourier-Stieltjes transforms on some finite interval. It is assumed here that either $G(x)$ has a derivative uniformly bounded in x , or $F(x)$ is a purely discontinuous function such that the functions $F(x)$ and $G(x)$ may have discontinuities only at points $x = x_\nu$ ($x_\nu < x_{\nu+1}$, $\nu = 0, \pm 1, \pm 2, \dots$), satisfying the condition $\min_\nu (x_{\nu+1} - x_\nu) \geq L > 0$, with $|G'(x)| \leq A$ everywhere, except at the points $x = x_\nu$. These results of Esseen can be encompassed by a single formulation. The following theorem is a generalization of Esseen's theorems.

Theorem 1. Let $F(x)$ be a nondecreasing function, and $G(x)$ a function of bounded variation on the real line, $F(-\infty) = G(-\infty)$, $F(+\infty) = G(+\infty)$. Let

$$f(t) = \int_{-\infty}^{\infty} e^{itx} dF(x), \quad g(t) = \int_{-\infty}^{\infty} e^{itx} dG(x),$$

and let T be an arbitrary positive number. Then, for any number $b > 1/2\pi$, the inequality

$$\sup_x |F(x) - G(x)| \leq b \int_{-T}^T \left| \frac{f(t) - g(t)}{t} \right| dt + bT \sup_x \int_{|y| \leq c(b)/T} |G(x+y) - G(x)| dy, \quad (1)$$

holds, where $c(b)$ is a positive constant depending only on b .

In inequality (1) one may take $c(b)$ equal to the root of the equation

$$\int_0^{1/4 c(b)} \frac{\sin^2 u}{u^2} du = \frac{\pi}{4} + \frac{1}{8b}.$$

In the special case when $F(x)$ and $G(x)$ are distribution functions, a similar result was obtained by A. C. Feinleib ⁽³⁾. A uniform estimate of the difference between a distribution function $F(x)$ and a certain function of bounded variation $G(x)$ that is not a distribution function is of considerable interest for applications (for example, in the study of asymptotic expansions in limit theorems for sums of independent random variables).

If the conditions of Theorem 1 are satisfied and the function $G(x)$ satisfies the following Lipschitz condition:

$$|G(x) - G(y)| \leq K|x - y|^\alpha$$

for all x and y and for some positive constants K and α , then the second term on the right-hand side of inequality (1) may be replaced by $2bK(c(b))^{1+\alpha}(1 + \alpha)^{-1}T^{-\alpha}$.

We give one more immediate consequence of Theorem 1, which is also a generalization of Esseen's theorem.

Theorem 2. Let $F(x)$ be a nondecreasing function, $G(x)$ a function of bounded variation, and let $f(t)$ and $g(t)$ be the corresponding Fourier-Stieltjes transforms; let T be an arbitrary positive number, and

$$F(-\infty) = G(-\infty), \quad F(+\infty) = G(+\infty).$$

Suppose $|G'(x)| \leq A$ everywhere, except at points of discontinuity of the function $G(x)$.

Then, for any number $b > 1/2\pi$, the inequality

$$\sup_x |F(x) - G(x)| \leq b \int_{-T}^T \left| \frac{f(t) - g(t)}{t} \right| dt + r(b) \frac{A}{T}, \quad (2)$$

holds, where $r(b)$ is a positive constant depending only on b .

In inequality (2) one may take $r(b) = bc^2(b)$, where $c(b)$ is the constant from Theorem 1.

Leningrad State University
named after A. A. Zhdanov

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REFERENCES

- ¹ C. G. Esseen, Acta Math., **77**, 1 (1945).
- ² B. V. Gnedenko, A. N. Kolmogorov, *Limit Distributions for Sums of Independent Random Variables*, Moscow-Leningrad, 1949.
- ³ A. C. Feinleib, Izv. Acad. Sci. USSR, Ser. Math., **32**, No. 4, 859 (1968).

Note: Figure translations are in progress. See original paper for figures.

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