

ON THE TOPOLOGY OF UNIVERSE MODELS

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Abstract

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MATHEMATICAL PHYSICS

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ON THE TOPOLOGY OF UNIVERSE MODELS

(Presented by Academician Ya. B. Zel'dovich, 24 IV 1970)

1°. An important question in cosmology is the problem of singling out those classes of manifolds that can serve as models of the Universe. For a correct consideration of Einstein's equations it is necessary first, proceeding from physical considerations, to choose the manifold on which they will be defined. On the basis of the most general physical laws, a number of principles have been proposed that restrict the topology of Universe models independently of their concrete form*.

($\mathfrak{B}$) $\mathfrak{M}^4$ is a smooth manifold on which one can define an indefinite metric of signature (1,3).

($\mathfrak{G}_1$) All timelike and isotropic vectors can be divided into two classes $\{+\}$ and $\{-\}$ in such a way that, if $\xi \in \{+\}$, then $-\xi \in \{-\}$, and conversely, and if $\xi_n \in \{+\}$ ($\{-\}$), then $\lim \xi_n \in \{+\}$ ($\{-\}$).

($\mathfrak{G}_2$) $\mathfrak{M}^4$ contains no closed timelike lines.

($\mathfrak{G}_3$) On $\mathfrak{M}^4$ there exists a smooth point function $f(P) = t$ such that in some local coordinates t coincides with the time coordinate.

(ZN) $\mathfrak{M}^4$ contains no nonorientable spaces $\mathfrak{M}^3$.

2°. The class $\mathfrak{B}$ is narrower than the class of all smooth manifolds, since an indefinite metric can be defined not on all manifolds (a necessary condition for this is equality to 0 of the Euler characteristic ⁽³⁾). The principle $\mathfrak{B}$ is the most fundamental of all these principles. The principles $\mathfrak{G}_1$ – $\mathfrak{G}_3$ were proposed by K. Gödel ⁽⁷⁾ as different formulations of the principle of causality. The principle ZN was proposed by Ya. B. Zel'dovich and I. D. Novikov as a restriction following from CP -noninvariance ^(1,2).

3°. Repeating literally the reasoning of ^(1,2), one can obtain a restriction on the topology of Universe models following from CT -noninvariance. If CP -noninvariance makes it possible to distinguish the two possible orientations of $\mathfrak{M}^3$, then CT -noninvariance distinguishes the two possible directions of time

at points of $\mathfrak{M}^3$, i.e., the two possible orientations of the timelike vectors defined on $\mathfrak{M}^3$. That is, upon transport along a closed contour on $\mathfrak{M}^3$, a timelike vector must not change its orientation, which coincides with the definition of two-sidedness of $\mathfrak{M}^3$ ⁽³⁾. Thus, the following restriction on the topology of Universe models is proposed:

($\mathfrak{M}\mathfrak{B}$) $\mathfrak{M}^4$ cannot contain one-sidedly embedded $\mathfrak{M}^3$'s.

Let us note that, if ZN is an internal restriction with respect to $\mathfrak{M}^3$, then $\mathfrak{M}\mathfrak{B}$ is an external one.

4°. Let us give an illustration of the use of the principles ZN and $\mathfrak{M}\mathfrak{B}$: we construct the manifold $\mathfrak{M}^4$ —the direct product of the Klein bottle with the plane $(p; r)$. Recall that the direct product of two manifolds with coordinates $(u; v)$ and $(p; r)$ is a manifold whose points have coordinates $(u; v; p; r)$. The Klein bottle is obtained by gluing, in a square in the plane $(u; v)$, two sides with preservation of orientation and two with reversal of it, as a result of which a coordinat—

* The following notation is used below: $\mathfrak{M}^4$ is space-time; $\mathfrak{M}^3$ is space (concerning this concept see §8); R^1 is the real line. The letters numbering the principles also denote the classes of manifolds singled out by them.

ian grid with “periodic” coordinates:

$$(u; v + 2\pi) = (u; v); \quad (u + 2\pi; 2\pi - v) = (u; v). \quad (1)$$

Let us specify on this $\mathfrak{M}^4$ two metrics:

$$ds^2 = dp^2 - du^2 - dv^2 - dr^2, \quad (2)$$

$$ds^2 = du^2 - dv^2 - dp^2 - dr^2. \quad (3)$$

In (2), $\mathfrak{M}^3$ is the direct product of a Klein bottle with R^1 —nonorientable—and (2) does not satisfy ZN ; in (3), $\mathfrak{M}^3$ is the product of a cylinder and a line—one-sidedly embedded in $\mathfrak{M}^4$ (for the proof see ⁽³⁾), and (3) does not satisfy $\mathfrak{M}\mathfrak{B}$. The idea of the examples is borrowed from ^(6, 8). The fact that (3) does not satisfy $\mathcal{G}_2$ is immaterial ⁽⁹⁾.

5°. Let us note that ZN and $\mathfrak{M}\mathfrak{B}$ are physically proved by direct experiments.

6°. What is the relation between the principles considered above? We shall show that, by strengthening $\mathcal{G}_3$, one can obtain a principle from which $\mathfrak{B}$, $\mathcal{G}_1$, $\mathcal{G}_2$, and $\mathfrak{M}\mathfrak{B}$ follow.

($\mathfrak{KB}$) On $\mathfrak{M}^4$ one can define a smooth function of a point $f(P) = t$ such that, in some local coordinates, t coincides with the time coordinate, t ranges over the interval (a, b) , and $f^{-1}(a) = \mathfrak{N}_1^{3*}$ and $f^{-1}(b) = \mathfrak{N}_2^3$ are three-dimensional closed smooth manifolds.

Then $f(P)$ is a Morse function of the manifold $\mathfrak{M}^4$ without critical points and $\mathfrak{N}_1^3$ ⁽⁴⁾ is diffeomorphic to $\mathfrak{N}_2^3$, and

$$\mathfrak{M}^4 = \mathfrak{N}^3 \times R^1. \quad (4)$$

As is easy to verify, the principles $\mathfrak{B}, \mathcal{G}_1, \mathcal{G}_2, \mathfrak{MB}$ are in this case trivially fulfilled, while ZN forbids nonorientability of $\mathfrak{N}^3$. That is, $\mathfrak{KB}$ reduces the problem of the topology of models of the Universe to the three-dimensional one, where it is already posed as the problem of different “gluings” of space, i.e., as the problem of Heegaard diagrams ⁽³⁾. Let us note that all cosmological models known to the author that have physical meaning, as well as solutions of the equations of the general theory of relativity, satisfy $\mathfrak{KB}$. It may be supposed that $\mathfrak{KB}$ follows from $\mathcal{G}_1 - \mathcal{G}_3, \mathfrak{B}$, and $\mathfrak{MB}$. It is also interesting to find out precisely which manifolds enter into the classes considered above.

7°. The principles formulated above admit a natural generalization to the case of arbitrary dimension and signature; here classes $\mathfrak{B}_p^i$ arise (i.e., the class of n -dimensional smooth manifolds on which one can specify a metric of signature $(p; q)$), etc. However, their intersection is nonempty only for some n, p, q , so that, perhaps, n, p , and q can be obtained from these principles. Let us note that if a violation of CPT were observed, it would not impose restrictions analogous to ZN and $\mathfrak{MB}$, since PT does not change the orientation of $\mathfrak{M}^4$.

8°. In conclusion we give various definitions of $\mathfrak{M}^3$:

($\mathfrak{C}$) $\mathfrak{M}^3$ is any spacelike three-dimensional submanifold embedded in $\mathfrak{M}^4$ ⁽⁷⁾.

($\mathfrak{GM}$) $\mathfrak{M}^3$ is a spacelike three-dimensional submanifold that every timelike geodesic intersects once and only once ^(5, 7).

In some works of Ya. B. Zel'dovich and A. L. Zel'manov, for $\mathfrak{M}^4$ representable in the form (4), the following definition of space is meant:

(ZZ) $\mathfrak{M}^3$ is a spacelike submanifold diffeomorphic to $\mathfrak{N}^3$.

* $f^{-1}(a) = \mathfrak{N}_1^3$ means that for all τ sufficiently close to a , $f^{-1}(\tau)$ is diffeomorphic to $\mathfrak{N}_1^3$.

For the definition G it has not been proved that at least one M^4 satisfies ZN ; the difficulties of GM are discussed in ⁽⁵⁾. In the case of justification of KP , the definition ZZ appears preferable to the author.

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