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**Abstract**

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**PHYSICS**

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## **THE PINCH EFFECT IN SEMICONDUCTORS UNDER CONDITIONS OF STRONG DEGENERACY OF AN ELECTRON-HOLE PLASMA**

*(Presented by Academician M. A. Leontovich, 9 VI 1969)*

In considering the pinch effect in a bipolar plasma, as a rule, one confines oneself to the model of a nondegenerate gas. The problem of the spatial distribution of carriers for this case was considered in works <sup>(1,2)</sup>. However, in the plasma of a solid the approximation of a nondegenerate gas is not always applicable. A case is quite realistic in which the current carriers (of one or both signs) are in a state close to complete degeneracy. Besides low temperatures and high concentrations (the latter are achieved, for example, by injection of carriers through contacts), a factor favoring degeneracy is the small effective mass of the carriers. Thus, in indium antimonide the effective mass of band electrons is 0.013 of the mass of free electrons. Another example is bismuth.

A numerical calculation of the spatial distribution of carriers in a degenerate pinch, carried out in work <sup>(3)</sup> for a crystal of cylindrical geometry, showed that as the current passing through the crystal increases, the plasma separates from the boundaries of the crystal. In the present work the equation of the degenerate pinch is investigated in a crystal having the form of a plate. It is shown that, for a certain value of a parameter proportional to the square of the carrier drift velocity, there exists a solution describing a plasma detached from the crystal boundaries.

If the effective masses of electrons  $m_n$  and holes  $m_p$  differ appreciably in magnitude and the criterion of strong degeneracy is fulfilled only for the lighter carriers, the structure of the bipolar-diffusion flow is determined mainly by the degenerate carriers, and the problem of the spatial distribution of carriers in the pinch effect practically does not differ from the problem in which carriers of both signs are strongly degenerate.

Let us consider a semiconductor in the form of an infinite plate ( $-d \leq y \leq d$ ); the total current is directed along the  $x$  axis. The geometry under consideration is an idealization of a crystal having the form of a parallelepiped, in which two

linear dimensions greatly exceed the third. It is assumed that the condition of quasineutrality is fulfilled—the local concentration of holes  $p$  is equal to the local concentration of electrons  $n$ . The spatial distribution of carriers in the crystal follows from the simultaneous solution of the continuity equations for electrons and holes and the stationary Maxwell equations. With the chosen geometry the problem reduces to a one-dimensional one—all quantities depend only on the coordinate  $y$ . The plate is assumed thin—its thickness is small or comparable with the bipolar diffusion length; the rate of surface generation-recombination is taken to be zero. In this case the spatial distribution of carriers is determined mainly by the balance of the diffusion and field flows of carriers transverse to the total current. The equation for the balance of volume generation and recombination is an additional condition to the equation for the change of the local carrier density.

We shall also adopt a number of simplifying assumptions: the drift velocity of electrons  $v_n$  greatly exceeds the drift velocity of holes  $v_p$ ; the time of re-

the relaxation of the carrier momentum does not depend on energy—there is no magnetoresistance (these approximations are essential only when considering the case in which the Larmor frequency of the more mobile carriers, associated with the intrinsic magnetic field of the current, is comparable in magnitude with the inverse momentum-relaxation time); the total number of carriers in the crystal does not depend on the magnitude of the current passing through the crystal.

In the indicated approximations, the continuity equation containing the intrinsic magnetic field of the current is reduced to the form

$$f^{\mu-1} \frac{df}{d\xi} + \alpha f(\xi) \int_0^\xi f(\eta) d\eta = 0 \quad (1)$$

with the supplementary condition

$$\int_0^1 f(\xi) d\xi = 1; \quad (2)$$

$\mu = 1$  if the electron gas is nondegenerate, and  $\mu = 5/3$  if the electrons are strongly degenerate.

Here

$$\xi = \frac{y}{d}; \quad f(\xi) = \frac{p(y)}{p_0}; \quad \alpha = \frac{4\pi e p_0}{\Delta} \left( \frac{v_n d}{c} \right)^2; \quad \Delta = \Delta_n + \Delta_p;$$

$p(y)$ ,  $p_0$  are the local and crystal-averaged densities of holes (electrons);  $\Delta_{n(p)} = kT/e$ , if the electrons (holes) are nondegenerate, and

$$\Delta_{n(p)} = \frac{\hbar^3 \pi^{4/3} 3^{-1/3} p_0^{1/3}}{em_{n(p)}},$$

if the electrons (holes) are degenerate;  $T$  is the carrier temperature;  $0 \leq \xi \leq 1$ —the distribution is symmetric with respect to the plane  $y = 0$ .

Introduce the function

$$u(\xi) = \int_0^\xi f(\eta) d\eta.$$

Then the first integral of equation (1) has the form:

$$\frac{\alpha\mu}{2} f^{-\mu}(0) u^2(\xi) = 1 - \left[ \frac{f(\xi)}{f(0)} \right]^\mu. \quad (3)$$

Since  $f(\xi) = du/d\xi$ , the function  $u(\xi)$  is found by simple integration and can be expressed through the hypergeometric function  $F$  as follows:

$$\xi = \int_0^{u(\xi)} \frac{d\zeta}{[f^\mu(0) - \frac{1}{2}\alpha\mu\zeta^2]^{1/\mu}} = \frac{u(\xi)}{f(0)} F\left(\frac{1}{\mu}, \frac{1}{2}; \frac{3}{2}; \frac{\alpha\mu}{2} f^{-\mu}(0) u^2(\xi)\right). \quad (4)$$

The constant  $f(0)$ —the dimensionless concentration of carriers at the center of the plate—is determined by means of condition (2), which in the notation adopted has the form

$$u(1) = 1.$$

Equations (3) and (4) jointly determine, in implicit form, the function  $f(\xi)$  (one may also use (4) and the condition  $f(\xi) = du/d\xi$ ). For  $\mu = 1$  (nondegenerate statistics), using the known Gauss formulas, we arrive at the result of works (2):

$$f(\xi) = f(0) \operatorname{ch}^{-2} \left( \xi \sqrt{\frac{1}{2}\alpha f(0)} \right). \quad (5)$$

Let us consider the question of the sign of the solution  $f(\xi)$ . To this end we use relation (3). If the hypergeometric series in (4) converges for

$$\frac{1}{2}\alpha\mu f^{-\mu}(0) u^2(\xi) = 1, \quad (6)$$

then this means, according to (3), that the function  $f$  vanishes at the point  $\xi$ . Hence—

It follows that, under the known conditions, the solution for the concentration ceases to be positive definite over the entire range of variation of  $\xi$ . Let us denote by  $\alpha_{\text{cr}}$  the value of  $\alpha$  for which condition (6) is satisfied at  $\xi = 1$ . Accordingly, for  $\alpha > \alpha_{\text{cr}}$  this condition is satisfied at  $\xi < 1$ . If, for  $1/2 \alpha \mu f^{-\mu}(0) u^2(1) = 1$ , the hypergeometric series in (4) diverges, the situation  $f(\xi) = 0$  is unattainable (there is no  $\alpha_{\text{cr}}$ ) and the solution is positive everywhere.

From the general theory of hypergeometric functions it follows that the hypergeometric series  $F(a, b; c; z)$  diverges at the point  $z = 1$  if  $\text{Re}(a + b - c) \geq 0$ . In the case considered here  $a + b - c = 1/\mu - 1$ . For  $\mu \leq 1$  the series diverges at the point  $z = 1$ . Thus, for nondegenerate statistics ( $\mu = 1$ ) the function  $f(\xi)$  does not go to zero. For  $\mu > 1$  (in the degenerate gas  $\mu = 5/3$ ) the series converges at the point  $z = 1$ . In this case the function  $f(\xi)$  can take a zero value.

The solution  $f(\xi) \leq 0$ , naturally, has no meaning. As  $f(\xi) \rightarrow 0$ , the carrier concentration is insufficient for their strong degeneracy, and the model adopted in this region is incorrect; here one should use the approximation of a nondegenerate gas. For applications one may use solution (4) in that range of  $\xi$  where  $f(\xi) \geq 0$ , setting this function equal to zero in the remaining part of the crystal. In this approximation the pinch layer in the plate has a sharp boundary—the plasma is separated from the crystal surface.

Imposing on equation (4) the condition  $f(1) = 0$  and using the relation  $F(a, b; c; 1) = \Gamma(c)\Gamma(c - a - b)/\Gamma(c - a)\Gamma(c - b)$ , we obtain the critical value of the parameter  $\alpha$ :

$$\alpha_{\text{cr}} = \frac{6}{5} \left[ \frac{\sqrt{\pi} \Gamma(0, 4)}{2 \Gamma(0, 9)} \right]^{5/3} \simeq 3,190. \quad (7)$$

For  $\alpha = \alpha_{\text{cr}}$  the constant

$$f(0) = \frac{\sqrt{\pi} \Gamma(0, 4)}{2 \Gamma(0, 9)} \simeq 1,840;$$

in the vicinity of the point  $\xi = 1$  the carrier distribution has the form:

$$f(\xi) = \frac{2^{1/2}(1 - \xi)^{3/2}}{5^{3/2}} \left[ \frac{\sqrt{\pi} \Gamma(0, 4)}{\Gamma(0, 9)} \right]^{5/2} \simeq 3,286(1 - \xi)^{3/2}.$$

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*Note: Figure translations are in progress. See original paper for figures.*

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