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Abstract

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MATHEMATICS

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APPLICATION OF THE CONTOUR INTEGRAL METHOD TO THE SOLUTION OF MULTIDIMENSIONAL MIXED PROBLEMS FOR PARABOLIC SYSTEMS OF SECOND ORDER

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In the present note the author gives an application of the contour integral method (according to the scheme of Chapters VIII–IX of the book ⁽¹⁾) to the solution of mixed problems for parabolic systems of second order containing, in the boundary condition, the derivative with respect to time in a domain D of three dimensions with boundary Γ .

Consider the system

$$M\left(t, \frac{\partial}{\partial t}\right)v = A(x)\Delta v + \sum_{i=1}^3 A_i(x)\frac{\partial v}{\partial x_i} + A_0(x)v + f(x, t) \quad (1)$$

with the boundary condition

$$\lim_{x \rightarrow z} \{[\alpha_0(z) + \alpha_1(z)M(t, \partial/\partial t)]dv(x, t)/dn_z + [\beta_0(z) + \alpha_1(z)\beta_1(z)M(t, \partial/\partial t)]v(x, t)\} = \psi(z, t), \quad z \in \Gamma, \quad (2)$$

and the initial condition

$$v(x, 0) = \Phi(x), \quad (3)$$

where $M(t, \partial/\partial t) = b_0(t)\partial/\partial t + b_1(t)$; $b_0(t)$, $b_1(t)$ are functions defined on the interval $[0, \infty)$; $A(x)$, $A_i(x)$ ($i = 0, 1, 2, 3$) are square matrices of order m , defined in the three-dimensional domain $D + \Gamma$.

It is assumed that the following conditions are fulfilled:

1°. In the domain $D + \Gamma$ the roots $\nu_i(x)$ ($i, \dots, m$) of the characteristic equation

$$\det(A(x) + \nu E) = 0$$

have constant multiplicity and strictly negative real parts. In addition, the functions $b_0(t)$, $b_1(t)$ are continuous on the interval $[0, \infty)$, and there exist positive numbers $\varepsilon > 0$, $C > 0$, $\sigma \geq 0$, $C_1 > 0$ such that the inequalities

$$0 < \varepsilon \leq b_0^{-1}(t) \leq Ce^{\sigma t}, \quad |b_1(t)| \leq C_1.$$

hold.

2°. The matrices $\alpha_k(z)$, $\beta_k(z)$ ($k = 0, 1$) are continuous on Γ , and for sufficiently large values of the complex parameter λ the matrix

$$[\lambda_0(z) + \lambda^2 \alpha_1(z)]^{-1} [\beta_0(z) + \lambda^2 \alpha_1(z) \beta_1(z)]$$

is bounded for $z \in \Gamma$ by a number independent of λ ; Γ is a Lyapunov surface; for $y \in \Gamma$ the vector-function $\psi(y, t)$ is an original in the sense of note (2).

3°. The vector-functions $\Phi(x)$, $\partial^k f(x, t)/\partial t^k$ ($k = 0, 1, 2$) have continuous derivatives with respect to all x_i ($i = 1, 2, 3$) in the domain $D + \Gamma$ for $t \geq 0$ and vanish in some boundary strip of the domain D .

In notes (3, 4) the existence of a positive δ was proved such that in the domain R_δ of values of λ satisfying the inequalities

$$|\lambda| \geq R, \quad |\arg \lambda| \leq \pi/4 + \delta, \quad (R_\delta)$$

there exists a solution, analytic in λ , of the corresponding spectral problem

$$A(x)\Delta u + \sum_{i=1}^3 A_i(x) \frac{\partial u}{\partial x_i} + (A_0(x) - \lambda^2)u = \Phi(x), \quad (4)$$

$$\lim_{x \rightarrow z} B(z, d/dn_z, \lambda^2)u(x, \lambda) = \tilde{\psi}(z, \lambda), \quad (5)$$

where $\tilde{\psi}(z, \lambda)$ is the analytic continuation of the vector function

$$\int_0^\infty \exp[-\lambda^2 t_1(t) + t_2(t)] t_1'(t) \psi(z, t) dt$$

to the whole domain R_δ ;

$$t_1(t) = \int_0^t b_0^{-1}(\tau) d\tau, \quad t_2(t) = \int_0^t b_0^{-1}(\tau) b_1(\tau) d\tau.$$

Let R be a sufficiently large positive number. Denote by S an infinite open contour lying in the domain R_δ , sufficiently remote parts of which (lying outside a circle with center at the origin and sufficiently large radius) coincide with the continuations of the rays $\arg \lambda = \pm(\pi/4 + \delta)$.

With the aid of the estimates obtained in Notes ^(3, 4), for the solution of the spectral problem, following the scheme of Chapter IX of the book ⁽¹⁾, the following assertions are proved:

Theorem 1. *Under conditions 1°–3° there exists a solution $v^{(1)}(x, t, \psi)$ of problem (1), (3), in the case $f(x, t) \equiv 0$, $\Phi(x) \equiv 0$, representable in the form of a contour integral*

$$v^{(1)}(x, t, \psi) = \frac{1}{\pi\sqrt{-1}} \int_S \exp[\lambda^2 t_1(t) - t_2(t)] \lambda u^{(1)}(x, \lambda, \tilde{\psi}) d\lambda, \quad (6)$$

where $u^{(1)}(x, \lambda, \tilde{\psi})$ is the solution of the spectral problem (4), (5) for $\Phi(x) \equiv 0$ in D .

Theorem 2. *Under the conditions of Theorem 1, if the vector functions $\Phi(k)$, $\partial^k f(x, t)/\partial t^k$ ($k = 0, 1, 2$) for $t \in [0, \infty)$ are continuously differentiable once in the domain $D + \Gamma$ and vanish in some boundary strip of the domain D , then problem (1)–(3) for $\psi(z, t) \equiv 0$ on Γ has a solution $v^{(2)}(x, t, \Phi, f)$, represented by the formula*

$$v^{(2)}(x, t, \Phi, f) = \frac{1}{\pi\sqrt{-1}} \int_S \lambda d\lambda \int_D G(x, \xi, \lambda) z(\xi, t, \lambda) dD_\xi, \quad (7)$$

where $G(x, \xi, \lambda)$ is the Green matrix of the spectral problem (4), (5); $z(x, t, \lambda)$ is the solution of the Cauchy problem

$$M(t, \partial/\partial t)z - \lambda^2 z = f(x, t), \quad z(0) = \Phi(x).$$

It is easy to verify that

$$z(\xi, t, \lambda) = \Phi(\xi) \exp[\lambda^2 t_1(t) - t_2(t)] + \\ + \int_0^t b_0^{-1}(\tau) f(\xi, \tau) \exp\{\lambda^2 [t_1(t) - t_1(\tau)] - [t_2(t) - t_2(\tau)]\} d\tau.$$

The integrals (6) and (7) can be calculated according to the scheme of Note ⁽⁵⁾, and then for $v^{(1)}$, $v^{(2)}$ we obtain a representation in the form of a series whose terms are expressed through the data of the problem.

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.

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