

ON A THEOREM OF M. V. KELDYSH AND M. A. LAVRENTIEV

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Abstract

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MATHEMATICS

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ON A THEOREM OF M. V. KELDYSH AND M. A. LAVRENTIEV

(Presented by Academician A. N. Tikhonov on October 6, 1969)

The central result of paper ⁽¹⁾ is Theorem 2, from which there immediately follows uniqueness (up to a constant) of the solution of the Neumann problem. This theorem is formulated as follows.

Let $u(M)$ be a function harmonic in some domain T of three-dimensional Euclidean space, not identically constant, and suppose that at a point M_0 of the boundary ∂T , $u(M)$ has a unique limiting value u_0 , equal to the lower bound of its values in T .

If in T one can inscribe a body congruent to the paraboloid $z_0 \geq z \geq \rho^{1+\alpha}$ ($\alpha > 0$; $\rho = \sqrt{x^2 + y^2}$) with vertex at M_0 , then

$$\lim_{r_{10} \rightarrow 0} \frac{u(M_1) - u_0}{r_{10}} > 0,$$

where M_1 is a point on the axis of the paraboloid, and r_{10} is the distance from M_1 to M_0 .

Let us now introduce the function $\varphi(t)$ ($t \in [0, t_0]$), satisfying the following conditions:

$$\varphi(t) \in C^{(1)}([0, t_0]) \cap C^{(\infty)}((0, t_0]); \quad (1)$$

$$\varphi(0) = \varphi'(0) = 0; \quad (2)$$

$$\varphi'(t) > 0, \quad \varphi''(t) > 0 \quad \text{in } (0, t_0); \quad (3)$$

$$\int_0^{t_0} \frac{\varphi(t) dt}{t^2} < \infty. \quad (4)$$

We shall call the body $z_0 \geq z \geq \varphi(\rho)$ ($\rho = (\sum_{i=1}^{n-1} x_i^2)^{1/2}$) a φ -paraboloid.

By constructing a lower barrier, one can prove that Theorem 2 remains valid in a domain $(T + \partial T)$ of n -dimensional Euclidean space whose boundary point M_0 can be touched from within by a φ -paraboloid. As such a barrier one may take the subharmonic function

$$\Psi_1(M) = z \exp \left[\lambda_1 \int_0^z \frac{\varphi(t) dt}{t^2} \right] - \lambda_2 \varphi(r),$$

where $r = \sqrt{\rho^2 + z^2}$, and λ_i (here and below, λ_i are positive constants).

At the same time, Theorem 2 fails in certain domains $(T + \partial T) \in A^{(1)}$.

Indeed, define a harmonic function $u(M)$ in a φ -paraboloid, where $\varphi(t)$ satisfies conditions (1)–(3), but

$$\int_0^{t_0} \frac{\varphi(t) dt}{t^2} = \infty.$$

Moreover, $u(M)$ must attain the minimal value u_0 in some neighborhood of M_0 . Then on the axis Oz

$$u(M_1) - u_0 \leq \lambda_3 z \exp \left[-\lambda_4 \int_z^c \frac{\varphi(t) dt}{t^2} \right].$$

As an upper barrier here one proposes the superharmonic function

$$\Psi_2(M) = \exp \left[-\lambda_4 \int_r^{r_0} \frac{\varphi(t) dt}{t^2} \right] (\lambda_5 z - \varphi(r)).$$

I express my sincere gratitude to the participants of A. N. Tikhonov's seminar for their attention to this work.

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REFERENCES

¹ M. V. Keldysh, M. A. Lavrent'ev, DAN, 16, No. 3 (1937).

Note: Figure translations are in progress. See original paper for figures.

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