

SPHERICAL WAVE OF A SCALAR PLANKEON

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Abstract

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PHYSICS

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SPHERICAL WAVE OF A SCALAR PLANKEON

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The basic equation for studying a spherical wave corresponding to a “scalar” plankeon has the form (1)

$$(g^{ik}\hat{p}_i\hat{p}_k + m^2c^2 + \hbar^2 R/6)\psi = 0. \quad (1)$$

The interval corresponding to the “Einstein universe” is

$$-ds^2 = -c^2d\tau^2 + \frac{dr^2}{1-r^2/a^2} + r^2d\Omega^2. \quad (2)$$

In this case

$$R = 6/a^2. \quad (3)$$

Since for the scalar wave function

$$g^{ik}\hat{p}_i\hat{p}_k = -\hbar^2 g^{ik}\nabla_i\nabla_k = -\hbar^2 g^{ik}\left(\frac{\partial^2}{\partial x^i\partial x^k} - \Gamma_{ik}^m\frac{\partial}{\partial x^m}\right),$$

equation (1) takes the form

$$\left[g^{ik}\left(\frac{\partial^2}{\partial x^i\partial x^k} - \Gamma_{ik}^m\frac{\partial}{\partial x^m}\right) - \left(\frac{m^2c^2}{\hbar^2} + \frac{1}{a^2}\right)\right]\psi = 0. \quad (4)$$

For the metric (2), the nonzero Christoffel symbols have the form

$$\Gamma_{11}^1 = \frac{r}{a^2(1-r^2/a^2)}, \quad \Gamma_{22}^1 = r\left(1 - \frac{r^2}{a^2}\right), \quad \Gamma_{33}^1 = r\sin^2\theta\left(1 - \frac{r^2}{a^2}\right),$$

$$\Gamma_{12}^2 = \Gamma_{13}^3 = 1/r, \quad \Gamma_{33}^2 = -\sin\theta\cos\theta, \quad \Gamma_{23}^3 = \text{ctg}\theta.$$

Now (4) can be written in explicit form

$$\left[-\frac{\partial^2}{c^2 \partial \tau^2} + \left(1 - \frac{r^2}{a^2}\right) \left(\frac{\partial^2}{\partial r^2} + \frac{2}{r} \frac{\partial}{\partial r} \right) - \frac{r}{a^2} \frac{\partial}{\partial r} + \frac{1}{r^2} \left[\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right] - \left(\frac{m^2 c^2}{\hbar^2} + \frac{1}{a^2} \right) \right] \psi = 0 \quad (5)$$

Since

$$\frac{\partial^2 \psi}{\partial r^2} + \frac{2}{r} \frac{\partial \psi}{\partial r} = \frac{1}{r} \frac{\partial^2 (r\psi)}{\partial r^2}, \quad r \frac{\partial \psi}{\partial r} = \frac{\partial (r\psi)}{\partial r} - \psi,$$

then (5) takes the form

$$\left[-\frac{\partial^2 (r\psi)}{c^2 \partial \tau^2} + \left(1 - \frac{r^2}{a^2}\right) \frac{\partial^2 (r\psi)}{\partial r^2} - \frac{r}{a^2} \frac{\partial (r\psi)}{\partial r} + \frac{r\psi}{a^2} + \frac{1}{r^2} \left[\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial (r\psi)}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2 (r\psi)}{\partial \varphi^2} \right] - \left(\frac{m^2 c^2}{\hbar^2} + \frac{1}{a^2} \right) \right] \psi = 0$$

or

$$\left[-\frac{\partial^2}{c^2 \partial \tau^2} + \left(1 - \frac{r^2}{a^2}\right) \frac{\partial^2}{\partial r^2} - \frac{r}{a^2} \frac{\partial}{\partial r} + \frac{1}{r^2} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right) - \frac{m^2 c^2}{\hbar^2} \right] \varphi = 0, \quad (6)$$

where $\varphi = \psi r$.

Set $r = a \sin \chi$; then (6) takes the form

$$\left[-\frac{\partial^2}{c^2 \partial \tau^2} + \frac{1}{a^2} \frac{\partial^2}{\partial \chi^2} + \frac{1}{a^2 \sin^2 \chi} \left(\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} \right) - \frac{m^2 c^2}{\hbar^2} \right] \varphi = 0. \quad (7)$$

The operator

$$\frac{1}{\sin \theta} \left[\frac{1}{\sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right) \right] + \frac{1}{\sin^2 \theta} \frac{\partial^2}{\partial \varphi^2} = -\hat{l}^2 = -l(l+1),$$

where $l = 0, 1, 2, \dots$ is the angular momentum (orbital quantum number).

Therefore (7) takes the form:

$$\left[-\frac{\partial^2}{c^2 \partial \tau^2} + \frac{1}{a^2} \frac{\partial^2}{\partial \chi^2} - \left(\frac{m^2 c^2}{\hbar^2} + \frac{l(l+1)}{a^2 \sin^2 \chi} \right) \right] \varphi = 0. \quad (8)$$

Let

$$\varphi = A(\chi) e^{-\frac{i}{\hbar} E \tau} y_{lm}(\theta; \varphi), \quad (9)$$

where

$$y_m = \theta_{lm} e^{im\varphi}; \quad (10)$$

$$\theta_{lm} = \frac{(-1)^{l+m}}{2^l l!} \sqrt{\frac{(2l+1)(l-m)!}{2(l+m)!}} \sin^m \theta \frac{d^{l+m} \sin^{2l} \theta}{(d \cos \theta)^{l+m}},$$

$m = 0, \pm 1, \pm 2, \dots$ is the magnetic quantum number.

Substituting (9) into (8), we obtain

$$\left[\left(\frac{E^2}{c^2} - m^2 c^2 \right) \frac{a^2}{\hbar^2} - \frac{l(l+1)}{\sin^2 \chi} \right] A + A_{\chi\chi} = 0. \quad (11)$$

Denote

$$(E^2/c^2 - m^2 c^2) a^2 / \hbar^2 = \xi^2 \quad (12)$$

and instead of $l(l+1)$ introduce $n(n-1) = l(l+1)$, whence $n_1 = l+1$, $n_2 = -l$; then (11) assumes the standard form

$$\sin^2 \chi A_{\chi\chi} + [\xi^2 \sin^2 \chi - n(n-1)] A = 0. \quad (13)$$

The general solution of equation (13), as is known, has the form ⁽²⁾

$$A = \sin^n \chi \left(\frac{1}{\sin \chi} D \right)^n (\bar{A}_1 e^{-i\xi\chi} + \bar{A}_2 e^{i\xi\chi}), \quad (14)$$

where

$$\left(\frac{1}{\sin \chi} D \right)^n = \frac{1}{\sin \chi} \frac{d}{d\chi} \left[\frac{1}{\sin \chi} \frac{d}{d\chi} \left(\frac{1}{\sin \chi} \frac{d}{d\chi} (\dots) \right) \right]. \quad (15)$$

Thus we find that

$$\psi = \frac{\bar{A}_1}{r} \theta_{lm} e^{-\frac{i}{\hbar} E \tau + im\varphi} \sin^{l+1} \chi \left(\frac{1}{\sin \chi} D \right)^{l+1} e^{-i\xi\chi} = \frac{\varphi}{r},$$

$$\psi^* = \frac{\bar{A}_2}{r} \theta_{lm}^* e^{\frac{i}{\hbar} E \tau - i m \varphi} \sin^{-l} \chi \left(\frac{1}{\sin \chi} D \right)^{-l} e^{-i \xi \chi} = \frac{\varphi^*}{r}, \quad (16)$$

where $\xi = pa/\hbar$, $p = \sqrt{E^2/c^2 - m^2 c^2}$ is the momentum.

For $l = 0$, $m = 0$ ($n = 0$) we shall have

$$\psi = \frac{A_1}{r} e^{-\frac{i}{\hbar} (E \tau + p a \chi)}, \quad \psi^* = \frac{A_2}{r} e^{\frac{i}{\hbar} (E \tau + p a \chi)}. \quad (17)$$

The trace of the energy-momentum tensor corresponds to the given wave equation (1) and has the form ⁽³⁾

$$-T = \frac{m^2 c^2}{\hbar^2} \langle \psi \psi^* \rangle. \quad (18)$$

If we choose $A_1 A_2 = \beta \frac{m^2 c^3 a^2}{\hbar}$, where $\beta = \text{const}$ is determined below, then $\nabla \psi \nabla \psi^*$ will have the dimension of energy density (as it should). In this case

$$\langle \psi \psi^* \rangle = \frac{8 \pi \beta m^2 c^3 a^2}{\pi^2 a^3 \hbar} \int_0^a \frac{dr}{\sqrt{1 - r^2/a^2}} = \frac{8 m^2 c^3 \beta}{\pi \hbar} \chi \Big|_0^{\pi/2} = \frac{4 \beta m^2 c^3}{\hbar}; \quad (19)$$

$$-T = 4 \beta m^4 c^5 / \hbar^3; \quad (20)$$

$$R = 6/a^2 = -\chi T = \frac{32 \pi \beta m^4 c}{\hbar^3} G. \quad (21)$$

If we define $-T = 2\varepsilon$ (with $3p + \varepsilon = 0$), which holds for the Einstein space with metric (2), then we find

$$-T = 3 m c^2 / 2 \pi a^3. \quad (22)$$

From (20) and (22) we have $m c a = \hbar (3/8 \pi \beta)^{1/3}$. Put $\beta = 3/8 \pi$; then

$$m c a = \hbar. \quad (23)$$

From (21) and (23) we have

$$m = \sqrt{\frac{c \hbar}{2G}}, \quad a = L = \sqrt{\frac{2G \hbar}{c^3}} = 2Gm/c^2 = r_g, \quad (24)$$

i.e., for m and a we obtain the Planck values (the values of the mass and size of the planckeon), with $a = L = r_g$ —the gravitational radius, as must be the case for an Einstein universe. At the same time, the result obtained checks the calculations performed.

Denote

$$\frac{1}{\hbar}[E\tau + pa\chi] = \frac{1}{\hbar} \left[E\tau + pa \arcsin \frac{r}{a} \right] = \alpha,$$

then (17), for $m = 0$, $l = 0$ ($n = 0$), takes the form

$$\psi = \frac{A_1}{r} e^{-i\alpha}. \quad (25)$$

The de Broglie wave will propagate according to the law:

$$E\tau + pa \arcsin \frac{r}{a} = \alpha\hbar,$$

or

$$r = a \sin \frac{E\tau - \alpha\hbar}{pa} = a \sin \frac{E(\tau - \tau_0)}{pa} = a \sin \left[\frac{E}{pc} \frac{c(\tau - \tau_0)}{a} \right]. \quad (26)$$

Hence the phase velocity is

$$\frac{V_{\text{ph}}}{c} = \sqrt{1 + \frac{m^2 c^2}{p^2}} \cos \left[\sqrt{1 + \frac{m^2 c^2}{p^2}} \frac{c(\tau - \tau_0)}{a} \right],$$

the group velocity is

$$\frac{V_{\text{gr}}}{c} = \frac{1}{\sqrt{1 + m^2 c^2 / p^2}}.$$

Expression (26) shows that the de Broglie wave corresponding to the planckeon is localized in the region $0 \leq r \leq a = r_g = L$ and that the wave packet, held by its own gravitational field, is stable and does not spread.

For values $c\tau/a \ll 1$,

$$r = c(\tau - \tau_0)E/pc = E(\tau - \tau_0)/p = c(\tau - \tau_0)\sqrt{1 + m^2 c^2 / p^2}, \quad (27)$$

i.e., for $a \rightarrow \infty$, when gravity is switched off, the usual-

...situation of spreading of the wave packet, since as $\tau \rightarrow \infty$, $r \rightarrow \infty$. The conclusion regarding the stability of the wave packet in its own gravitational field of a definite energy is fundamental, although quite expected. This conclusion is significant in studying the structure of elementary particles in their own gravitational field.

Earlier M. A. Markov ⁶ proposed that the maximon (a particle equivalent to the planckon) is described by a particle-like solution of the Einstein and Dirac equations in the form of a limiting state of a wave packet whose energy is gravitationally closed in a region of size L .

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CITED LITERATURE

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