

ON THE QUESTION OF THE KINETIC DESCRIPTION OF RAPID PROCESSES IN A PLASMA

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Abstract

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PHYSICS

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ON THE QUESTION OF THE KINETIC DESCRIPTION OF RAPID PROCESSES IN A PLASMA

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The state of a plasma is usually described by the distribution function $f(t, \mathbf{r}, \mathbf{v})$, which is the probability density of the random coordinates $\mathbf{r}(t)$ and velocity $\mathbf{v}(t)$ of a particle. The distribution function depends on time t as on a parameter and satisfies a kinetic equation of the Fokker–Planck type

$$\frac{\partial f}{\partial t} = -v_i \frac{\partial f}{\partial x_i} - \frac{\partial}{\partial v_i} (A_i f) + \frac{1}{2} \frac{\partial^2}{\partial v_i \partial v_j} (D_{ij} f). \quad (1)$$

On the other hand, it is known from the theory of random processes [1] that an equation of type (1) exists only for Markov random processes $\{\mathbf{r}(t), \mathbf{v}(t)\}$, for which the rate of change of the distribution function $\partial f / \partial t$ at time t depends only on the value of f at this time and does not depend on the preceding history, i.e., on the values of f at preceding instants of time. In particular, the process $\{\mathbf{r}(t), \mathbf{v}(t)\}$ will be non-Markovian if $\{\mathbf{r}(t), \mathbf{v}(t)\}$ is the solution of a system of differential equations whose coefficients are random functions with a nonzero correlation time.

In the case of a plasma the state of a particle with mass m and charge e is determined by the equations

$$\frac{d\mathbf{r}}{dt} = \mathbf{v}, \quad \frac{d\mathbf{v}}{dt} = \frac{e}{m} \mathbf{E}(\mathbf{r}, t), \quad (2)$$

where the electric field $\mathbf{E}$ is a random process with a correlation function that may be approximated by the exponential

$$\langle E_i[\mathbf{r}(t), t] E_j[\mathbf{r}(t + \tau), t + \tau] \rangle = \delta_{ij} \sigma^2 e^{-\nu|\tau|}. \quad (3)$$

The correlation time $1/\nu$ usually has the order of the period of plasma oscillations.

To reveal the main features of this random process, we shall consider the simplest case of one-dimensional homogeneous (i.e., independent of $\mathbf{r}$) motion. Then equations (2), (3) are simplified:

$$\dot{v} = \frac{e}{m}E, \quad (4)$$

$$\langle E(t)E(t+\tau) \rangle = \sigma^2 e^{-\nu|\tau|}. \quad (5)$$

The process $v(t)$ is non-Markovian, but it can be augmented to a Markovian one [1]. To do this, using the Wiener–Khinchin theorem, we transform relation (5) to the form

$$\dot{E} + \nu E = u(t), \quad (6)$$

where $u(t)$ is white noise having zero correlation time

$$\langle u(t)u(t+\tau) \rangle = 2\sigma^2\nu\delta(\tau). \quad (7)$$

The two-dimensional random process $\{v(t), E(t)\}$, described by equations (4), (6), is Markovian, and its distribution function $f(v, E)$ satisfies the Fokker–Planck equation (1)

$$\frac{\partial f}{\partial t} = -\frac{e}{m}E \frac{\partial f}{\partial v} + \nu \frac{\partial(fE)}{\partial E} + \sigma^2\nu \frac{\partial^2 f}{\partial E^2}. \quad (8)$$

If the electric field had no “memory,” i.e., if it itself were white noise, then the distribution function would depend only on the velocity: $f(v)$. (The distribution function has the same form after the lapse of a time much longer than the correlation time. Such a consideration is used in the theory of stochastic acceleration of particles ^(2,3).) The simplest “memory” of the type (5) leads to the distribution function $f(v, E)$, or, by virtue of (4), $f(v, \dot{v})$. More complicated correlation functions of the electric field will lead to a dependence of the distribution functions on higher derivatives of the velocity:

$$f(v); f(v, \dot{v}); f(v, \dot{v}, \ddot{v}); f(v, \dot{v}, \ddot{v}, \ddot{\ddot{v}}), \dots \quad (9)$$

Such a chain of one-particle distribution functions, depending on the velocity v and its derivatives (this possibility was first pointed out by Vlasov ⁽⁴⁾), is analogous to Bogolyubov’s chain ⁽⁵⁾ $f(v_1), f(v_1, v_2), f(v_1, v_2, v_3), \dots$ of many-particle distribution functions depending only on the velocity. The chain (9) may prove preferable in the case of strongly turbulent plasma, when the correlations of the distribution functions $g(v_1, v_2) = f(v_1, v_2) - f(v_1)f(v_2)$ are not small ⁽⁶⁾.

(At present, a quantitative theory of strongly turbulent plasma has not yet been created ⁽⁷⁾.)

It can be shown that in a quiescent plasma the velocity of particles is a Markov process, if one neglects terms that are small in comparison with the Coulomb logarithm.

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