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Abstract

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HYDROMECHANICS

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ON COLLISIONS OF SOLID PARTICLES IN A LIQUID

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The study of the laws of collision of solid bodies in a liquid or gas is of great interest for the analysis of particle interactions in a fluidized (boiling) bed. The traditional relations in this case can be applied only as a certain approximation, since the dynamic properties of the liquid influence the nature of the process and the very outcome of the collision.

In the present work we study the process of binary impacts in a system of N spheres immersed in an ideal incompressible liquid at rest at infinity. We denote by a the radius of the spheres and by m their mass. We shall assume that the motion of the liquid is caused by the imposition of a certain field of impulsive pressures, and that the interaction potential between the particles is pairwise. Within the framework of the assumptions made, the motion of the system of spheres together with the liquid is described by a finite number of explicit coordinates ⁽¹⁾: the radius vectors $\mathbf{r}_i$ of the centers of the particles relative to some fixed Cartesian coordinate system. In the absence of other acting forces there is an energy integral

$$E = \frac{1}{2} \sum_{i,j=1}^N \sum_{\alpha,\beta=1}^3 \left(m \delta_{ij} \delta_{\alpha\beta} + \frac{2}{3} \pi \rho a^3 a_{ij}^{\alpha\beta} \right) u_{i\alpha} u_{j\beta} + \frac{1}{2} \sum_{\substack{i,j=1 \\ (i \neq j)}}^N U(|\mathbf{r}_i - \mathbf{r}_j|) = \text{const.} \quad (1)$$

Here $u_{i\alpha}$ ($i = 1, 2, \dots, N$; $\alpha = 1, 2, 3$) are the components of the velocity vector of the i -th sphere; ρ is the density of the liquid, and the coefficients $a_{ij}^{\alpha\beta}$ are expressed in a known way through the potential of the velocity field.

The interaction potential of solid spheres may be represented in the form

$$U(s) = \begin{cases} 0, & \text{if } s > 2a, \\ \infty, & \text{if } s < 2a. \end{cases}$$

Introduce the notation

$$D = \det \|A_{ij}^{\alpha\beta}\|, \quad A_{ij}^{\alpha\beta} \equiv m\delta_{ij}\delta_{\alpha\beta} + 2/3\pi\rho a^3 a_{ij}^{\alpha\beta}.$$

Let $D_{ij}^{\alpha\beta}$ be the algebraic cofactor of the element $A_{ij}^{\alpha\beta}$, and

$$K = 1/2 \sum_{i,j=1}^N \sum_{\alpha,\beta=1}^3 A_{ij}^{\alpha\beta} u_{i\alpha} u_{j\beta}.$$

The time evolution of the system is described by Lagrange equations of the second kind with the Lagrangian

$$L = K - 1/2 \sum_{\substack{i,j=1 \\ (i \neq j)}}^N U(|\mathbf{r}_i - \mathbf{r}_j|).$$

Solving the Lagrange equations with respect to the highest derivatives, we obtain

$$\frac{du_{i\alpha}}{dt} = - \sum_{\substack{k,j=1 \\ (k \neq j)}}^N \sum_{\beta=1}^3 \frac{D_{ij}^{\alpha\beta}}{D} \frac{\partial U(|\mathbf{r}_k - \mathbf{r}_j|)}{\partial r_{j\beta}} + \sum_{k=1}^N \sum_{\gamma=1}^3 \frac{D_{ik}^{\alpha\gamma}}{D} \left(\frac{\partial K}{\partial r_{i\alpha}} - \sum_{n=1}^N \sum_{\omega=1}^3 u_{n\omega} \frac{\partial^2 K}{\partial u_{k\gamma} \partial r_{n\omega}} \right). \quad (2)$$

Suppose that at the time $t = 0$ a collision of two spheres occurs, while all the others at that instant are at a finite distance both from the colliding pair and from one another. Without loss of generality, the particles may be numbered so that the numbers of the colliding particles are 1 and 2. Denote

$$\sigma = \mathbf{r}_1 - \mathbf{r}_2, \quad \mathbf{l} = \sigma / \sigma.$$

Taking into account the assumption made about the configuration of the system of spheres at the time $t = 0$, integrate (2) with respect to time from $t = -\varepsilon$ to $t = \varepsilon$ and pass to the limit as $\varepsilon \rightarrow 0$. The terms containing K in (2), after passage to the limit, vanish by virtue of their boundedness, so that as a result we shall have

$$\Delta u_{j\beta} = \sum_{\alpha=1}^3 l_{\alpha} \frac{D_{2j}^{\alpha 3} - D_{1j}^{\alpha\beta}}{D} I, \quad I = \lim_{\varepsilon \rightarrow 0} \int_{-\varepsilon}^{\varepsilon} \frac{\partial U(\sigma)}{\partial \sigma} dt. \quad (3)$$

The coefficients of I in (3) are taken at the time $t = 0$. Using now the energy integral (1), we find

$$I = 2D \sum_{\alpha=1}^3 l_{\alpha} (u_{1\alpha} - u_{2\alpha}) / \sum_{\alpha,\beta=1}^3 l_{\alpha} l_{\beta} (D_{11}^{\alpha\beta} + D_{22}^{\alpha\beta} - D_{12}^{\alpha\beta} - D_{21}^{\alpha\beta}). \quad (4)$$

Substituting (4) into (3), after simple transformations we obtain

$$\Delta u_{j\beta} = m_{j\beta} (\mathbf{l} \cdot \mathbf{g}), \quad \mathbf{g} = \mathbf{u}_1 - \mathbf{u}_2,$$

$$m_{j\beta} = 2 \sum_{\alpha=1}^3 l_{\alpha} (D_{2j}^{\alpha\beta} - D_{1j}^{\alpha\beta}) / \sum_{\alpha,\beta=1}^3 l_{\alpha} l_{\beta} (D_{11}^{\alpha\beta} + D_{22}^{\alpha\beta} - D_{12}^{\alpha\beta} - D_{21}^{\alpha\beta}). \quad (5)$$

As $\rho \rightarrow 0$, it follows from (5) that $m_{j\beta} = 0$ ($j \geq 3$), $m_{1\beta} = -m_{2\beta} = l_{\beta}$. Relations (5) represent the solution of the problem of the collision of two spherical particles in an ideal incompressible fluid. The laws of collision turn out to differ from the traditional ones primarily in that, in a binary collision, the velocities of all particles of the system change discontinuously. If one separately traces the energies of the system of particles and of the fluid, it is easy to see that as a result of the collision a discontinuous redistribution of the total energy of the system occurs. In a completely analogous way it can be shown that the momentum of the colliding particles is also not conserved.

For a quantitative estimate of the magnitude of the defect of momentum and energy of the colliding particles, we use the formulas for $a_{ij}^{\alpha\beta}$ obtained in (2) with accuracy up to terms of order $(a/L)^6$, where L is the mean distance between particles:

$$a_{ij}^{\alpha\beta} = \delta_{ij} \delta_{\alpha\beta} - \frac{3a^2}{2} \Lambda_{ij}^{\alpha\beta} - \frac{3a^6}{4} \sum_{k=1}^N \sum_{\gamma=1}^3 \Lambda_{ik}^{\alpha\gamma} \Lambda_{kj}^{\gamma\beta},$$

$$\Lambda_{ij}^{\alpha\beta} = \frac{3r_{ij}^{\alpha} r_{ij}^{\beta}}{r_{ij}^5} - \frac{\delta_{\alpha\beta}}{r_{ij}^3}, \quad \Lambda_{ii}^{\alpha\beta} \equiv 0, \quad r_{ij} = |\mathbf{r}_i - \mathbf{r}_j|.$$

Introducing the effective mass of a particle in a liquid, $m_0 = m + \frac{2}{3}\pi\rho a^3$, we represent $A_{ij}^{\alpha\beta}$ in the form

$$A_{ij}^{\alpha\beta} = m_0 (\delta_{ij} \delta_{\alpha\beta} - \varepsilon b_{ij}^{\alpha\beta}), \quad \varepsilon = \chi / (1 + \chi), \quad \chi = \rho / 2\rho_p,$$

$$b_{ij}^{\alpha\beta} = \frac{3}{2} a^3 \Lambda_{ij}^{\alpha\beta} + \frac{3}{4} a^6 \sum_{k=1}^N \sum_{\gamma=1}^3 \Lambda_{ik}^{\alpha\gamma} \Lambda_{kj}^{\gamma\beta}, \quad (6)$$

where ρ_p is the density of the particle material.

By definition,

$$\Delta E_{12} = \frac{m_0}{2} [(\mathbf{u}_1 + \Delta \mathbf{u}_1)^2 + (\mathbf{u}_2 + \Delta \mathbf{u}_2)^2 - \mathbf{u}_1^2 - \mathbf{u}_2^2] = m_0 \varepsilon \sum_{i,j=1}^2 \sum_{\alpha,\beta=1}^3 b_{ij}^{\alpha\beta} u_{i\alpha} \Delta u_{j\beta} + \frac{1}{2} m_0 \varepsilon \sum_{i,j=1}^2 \sum_{\alpha,\beta=1}^3 b_{ij}^{\alpha\beta} \Delta u_{i\alpha} \Delta u_{j\beta}$$

Similarly, the momentum defect of the colliding particles is, by definition, equal to

$$\Delta p_\alpha = m_0 (\Delta u_{1\alpha} + \Delta u_{2\alpha}) = 2m_0 (\mathbf{l} \mathbf{g}) \sum_{\gamma=1}^3 l_\gamma (D_{22}^{\alpha\gamma} - D_{11}^{\alpha\gamma}) / \sum_{\alpha,\beta=1}^3 l_\alpha l_\beta (D_{11}^{\alpha\beta} + D_{22}^{\alpha\beta} - D_{12}^{\alpha\beta} - D_{21}^{\alpha\beta}). \quad (8)$$

In (7), the first two terms on the right-hand side are directly connected with the colliding particles, and for the computation of $A_{ij}^{\alpha\beta}$ ($i, j \leq 2$) formulas (6) are not valid. For all the other terms on the right-hand side of (7), these formulas make it possible to estimate their dependence on the distance to the place of collision.

Let us compute the cofactors of the elements of the determinant D . It follows from (6) that D is a function of ε , and since even for $\rho = \rho_p$ the value of the parameter ε does not exceed $1/3$, it is convenient to represent the expressions for D and for the cofactors of its elements in the form of expansions in powers of the small parameter.

Restricting ourselves to terms of order ε , we obtain

$$\frac{1}{m_0^{3N-1}} (D_{2j}^{\alpha\beta} - D_{1j}^{\alpha\beta}) = \varepsilon (-1)^{3(j-1)+\alpha+\beta} (b_{2j}^{\alpha\beta} - b_{1j}^{\alpha\beta}). \quad (9)$$

If we denote by $\mathbf{R}$ the radius vector of the point of contact of the spheres at the time $t = 0$, then

$$\mathbf{r}_1 = \mathbf{R} + a\mathbf{l}, \quad \mathbf{r}_2 = \mathbf{R} - a\mathbf{l}, \quad \mathbf{s}_j = \mathbf{R} - \mathbf{r}_j$$

and, as follows from (6) and (9),

$$\frac{1}{m_0^{3N-1}} (D_{2j}^{\alpha\beta} - D_{1j}^{\alpha\beta}) = \varepsilon k_j^{\alpha\beta} \left(\frac{a}{s_j} \right)^4,$$

where $k_j^{\alpha\beta}$ are functions of the direction cosines of the vectors $\mathbf{s}$ and $\mathbf{l}$.

The characteristic magnitude of the jumps in the velocities of the particles can now be obtained from (5):

$$\Delta u_{j\beta} = \left[2\varepsilon(\mathbf{lg}) \sum_{\alpha=1}^3 l_{\alpha} k_j^{\alpha\beta} / \sum_{\alpha,\beta=1}^3 l_{\alpha} l_{\beta} (D_{11}^{\alpha\beta} + D_{22}^{\alpha\beta} - D_{12}^{\alpha\beta} - D_{21}^{\alpha\beta}) \right] \left(\frac{a}{s_j} \right)^4. \quad (10)$$

The magnitude of these jumps rapidly decreases with increasing distance from the place of collision.

From relations (7) and (10) one may conclude that the last four terms in the expression for the energy defect in a collision will make a small contribution to the right-hand side of (7). Indeed, all the terms indicated above will be of order no less than $\varepsilon(a/L)^4$.

For $\varepsilon \ll 1$, i.e., in gas flows, binary collisions of solid particles may, to a high degree of accuracy, be regarded as satisfying the classical relations.

In drop liquids, when $\varepsilon \simeq 1/3$, the classical relations are no longer applicable. Taking into account the dependence of the right-hand side in (7) on (a/L) , one may neglect the last 4 terms in this equality. The laws of collision will have, as in the classical case, a local structure, but between the particles and the liquid at the instant of collision there will be an exchange of momentum and energy.

The range of applicability of the model under consideration to real systems is determined by the compressibility of the liquid and of the particle material. Denoting the speed of sound in the liquid by c , and by c_0 the speed of sound in the particle material, the condition for applicability of the model may be written in the form $(ca/c_0L) \gg 1$.

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.

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