
Soviet-era science, translated into English

THE PROBLEM OF THE ESCAPE OF ONE CONTROLLED OBJECT FROM ANOTHER

L. S. Pontryagin, E. F. Mishchenko

1969

SovietRxiv

View the original and related papers at <https://sovietrxiv.org/items/ru-196901.52044>

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

Abstract

Full Text

UDC 517.92

MATHEMATICS

Academician L. S. PONTRYAGIN, E. F. MISHCHENKO

THE PROBLEM OF THE ESCAPE OF ONE CONTROLLED OBJECT FROM ANOTHER

In recent years many works have appeared (see, for example, ⁽¹⁻⁵⁾) devoted to the problem of one controlled object

$$\dot{x} = f(x, u) \quad (1)$$

pursuing another controlled object

$$\dot{y} = g(y, v) \quad (2)$$

Here x, y are the phase vectors of the objects, and u, v are the control parameters.

In this problem the active role is assigned to the pursuing parameter u , whose goal is to bring the spatial coordinates of the objects (1) and (2) into coincidence as quickly as possible, under an arbitrary control v of object (2).

However, another problem is just as natural—that of the escape of object (2) from object (1). In its formulation, the active role is assigned to the escaping parameter v , whose goal is to prevent the coincidence of the spatial coordinates of the objects (1) and (2) for any value of time t , under an arbitrary control u of object (1).

In the present note a linear differential game is considered from the point of view of the problem of escape.

The equation of the game is the following:

$$\dot{z} = Cz - u + v, \quad (3)$$

where z is the phase vector of the n -dimensional Euclidean space R^n ; C is a constant square matrix; u is the pursuing parameter; v is the escaping parameter; $u \in P$, $v \in Q$; P and Q are convex compact subsets of the space R^n of arbitrary dimensions, generally speaking less than n .

We shall say that escape is possible in the game (3) if, for any initial value $z_0 \in R^n$, $z_0 \in M$ of the vector z , and for an arbitrary variation of the control

parameter $u = u(t)$, it is possible to choose such a variation of the control parameter $v = v(t)$ that the point $z(t)$ does not fall on the set M for any value of time t . At the same time, in order to find the value of the parameter $v(t)$ at each instant of time t , it is permitted to use only the values $u(t)$ and $z(t)$ at the same instant of time, and it is not permitted to use the values of u at times subsequent to t .

In the present note we give a condition sufficient for the possibility of escape in the game (3). At the same time a lower estimate is given for the distance of the point $z(t)$ from the subspace M . The latter is very substantial.

Let us proceed to the formulation of the result.

By L we denote the orthogonal complement to M in the space R^n . Let the dimension of L be ν , and suppose that

$$\nu \geq 2. \quad (4)$$

By π we denote the operation of orthogonal projection from R^n onto L . The sets

$$\pi C^{iP}, \quad \pi C^{iQ}, \quad i = 0, 1, 2, \dots, \quad (5)$$

are compact convex subsets of the space L .

Our main assumption is the following.

A. There exists a positive integer k such that, for $i < k - 1$, the sets (5) are points; the set $\pi C^{k-1}Q$ has dimension ν , and the set πC^kP can be carried by translation strictly inside the set $\pi C^{k-1}Q$.

Now we can formulate our main result.

Theorem. *If condition A and inequality (2) are satisfied in the differential game (3), then evasion is possible; moreover, there exists a positive number ε such that if*

$$|\pi z(0)| < \varepsilon, \quad (6)$$

then the following lower estimate holds for the distance $|\pi z(t)|$ of the point $z(t)$ from M for all $t \geq 0$:

$$|\pi z(t)| \geq \gamma(|z(t)|) |\pi z(0)|^k, \quad (7)$$

where γ is a monotonically decreasing function of its argument, depending only on the game and not on the initial value.

It is obvious that the possibility of evasion already follows from inequality (7).

Of course, condition A is not necessary for evasion to be possible in every game of the form (3). However, this condition is satisfied in a number of concrete nontrivial examples. Let us indicate, in particular, the example

$$\begin{aligned}\ddot{x} + \alpha \dot{x} &= \rho u, \\ \ddot{y} + \beta \dot{y} &= \sigma v.\end{aligned}\tag{8}$$

Here x, y, u, v are vectors of some Euclidean space E of dimension ≥ 2 ; x is the position of the pursuing point; y is the position of the pursued point; u and v are control parameters; $|u| \leq 1$; $|v| \leq 1$; $\alpha, \beta, \sigma, \rho$ are positive numbers. Pursuit is considered completed at the moment when $x = y$.

To pass from this pursuit problem to a differential game, put

$$z = (x - y, \dot{x}, \dot{y}).\tag{9}$$

The manifold M is defined here by the condition $x - y = 0$. It is easy to see that, for this differential game, condition A is satisfied if $\sigma > \rho$, with $k = 2$.

Owing to the presence of friction, $\alpha > 0$, $\beta > 0$, in motion starting from rest ($\dot{x} = 0, \dot{y} = 0$), the velocities $\dot{x}(t), \dot{y}(t)$ will be bounded:

$$|\dot{x}(t)| \leq \rho/\alpha, \quad |\dot{y}(t)| \leq \sigma/\beta,\tag{10}$$

the projection $(0, \dot{x}(t), \dot{y}(t))$ of the point $z(t)$ onto M will remain throughout within the compact set (10), and inequality (7) for our game will be written in the form

$$|\pi z(t)| \geq c |\pi z(0)|^2,\tag{11}$$

where c is a constant.

Condition A is also satisfied in the “boy and crocodile” problem, which is described by the system of differential equations

$$\begin{aligned}\ddot{x} &= u, \\ \ddot{y} &= v,\end{aligned}\tag{12}$$

where x, y, u, v are vectors of some Euclidean vector space E of dimension ≥ 2 , x is the vector determining the position of the pursuing object (the “crocodile”), y is the vector determining the position of the pursued object (the “boy”),

and pursuit is considered completed at the moment when $x = y$. It is easy to see—

that in the game corresponding to this problem, condition A is satisfied, with $k = 1$.

In conclusion we make one essential remark. The formulation of the evasion problem that we gave above is not entirely realistic. We assumed that, in the process of evasion, object (2) continuously observes object (1) and at each instant of time chooses the value of the control parameter t in order to escape from object (1). The future behavior of object (1) is not assumed to be known. However, the assumption that the state of object (1) is completely known at the present instant of time is unrealistic. Only the spatial coordinates of object (1) can be observed directly, while to obtain the entire vector x differentiation is necessary, and further operations are needed to obtain the parameter u ; moreover, processing all the information also requires some time. Thus, at the instant of time t we can use information about the object x referring only to the time $t - \delta$, where $\delta > 0$ is a small positive number. From the results formulated above there follows the possibility of evasion of the object $y(t)$ from the object $x(t - \delta)$, and estimate (7) makes it possible to estimate the distance between the objects $x(t)$ and $y(t)$, provided only that δ is sufficiently small.

Steklov Mathematical Institute
Academy of Sciences of the USSR
Moscow

Received
15 IX 1969

REFERENCES

1. L. S. Pontryagin, *UMN*, **21**, issue 4 (130), 219 (1966).
2. E. F. Mishchenko, L. S. Pontryagin, *DAN*, **174**, No. 1, 27 (1967).
3. L. S. Pontryagin, *DAN*, **174**, No. 6, 1278 (1967).
4. L. S. Pontryagin, *DAN*, **175**, No. 4, 764 (1967).
5. B. N. Pshenichnyi, *Automation and Remote Control*, No. 1, 65 (1968).

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.