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Abstract

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PHYSICS

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RAYLEIGH WAVES IN SEMICONDUCTING PIEZOELECTRIC CRYSTALS OF GALLIUM ARSENIDE

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It is known that in two groups of semiconducting piezocrystals an intense interaction of ultrasonic waves with the conduction electrons of the crystal is observed. The first group, A_2B_6 , includes crystals of CdS, CdSe, ZnO, etc.; the second group, A_3B_5 , includes crystals of GaAs, InSb, InAs, etc. The propagation of bulk (longitudinal and transverse) waves in such crystals and their interaction with conduction electrons have been studied quite thoroughly (^{1, 2}). The propagation of surface Rayleigh waves in these crystals (with account taken of piezoelectric properties and conductivity) has been studied only for crystals of the first group, mainly for CdS (³⁻⁵). In the present work an attempt is made to solve the problem of the propagation of plane harmonic Rayleigh waves in a crystal of the second group—gallium arsenide (hexatetrahedral symmetry class).

As shown in (³), a Rayleigh wave propagating in an arbitrary direction in a semiconducting piezoelectric crystal is described by the system of equations:

$$\rho \partial^2 U_i / \partial t^2 = \partial T_{ik} / \partial x_k; \quad (1)$$

$$T_{ik} = c_{iklm}^E U_{lm} - e_{jik} E_j; \quad (2)$$

$$D_n = 4\pi e_{nlm} U_{lm} + \varepsilon_{jn}^S E_j; \quad (3)$$

$$\partial D_n / \partial x_n = -4\pi en; \quad (4)$$

$$j_k = \sigma E_k + efD \partial n / \partial x_k; \quad (5)$$

$$\partial j_k / \partial x_k = e \partial n / \partial t. \quad (6)$$

Here ρ is the density of the crystal; U_i are the components of displacement in the wave; T_{ik} , U_{lm} , c_{iklm}^E are the components of the tensors, respectively, of stress, strain, and elastic moduli (the latter taken at constant electric field); $E_j = -\partial\psi/\partial x_j$ and D_n are the components of the vectors of electric field and induction; ψ is the electric potential; ε_{jn}^S are the components of the dielectric-permittivity tensor at constant entropy; e_{jik} are the components of the piezoelectric-constant tensor; j_k are the components of current; D and σ are the electron diffusion coefficient and the conductivity of the crystal, with $\sigma = e\mu(n_0 + fn)$, $\sigma_0 = e\mu n_0$; e is the electron charge, μ is the electron mobility, n is the excess of the electron concentration over its equilibrium value n_0 (in the absence of the wave); f is the trap factor.

At the boundary of the crystal with vacuum (a plane with normal $\mathbf{m}$), along which the Rayleigh wave propagates, the boundary conditions of absence of stresses and continuity of the tangential component of the electric-field vector must be satisfied:

$$T_{im} = 0; \quad (7)$$

$$(E_\tau)_{\text{cr}} = (E_\tau)_{\text{vac}}. \quad (8)$$

To derive the last boundary condition, let us turn to the fundamental equations (4)–(6). They can be reduced to a single equation of the form

$$\frac{\partial}{\partial x_k}(\sigma E_k) - \frac{1}{4\pi} \frac{\partial}{\partial x_k} \left(f D \frac{\partial^2 D_i}{\partial x_k \partial x_i} \right) + \frac{1}{4\pi} \frac{\partial^2 D_k}{\partial x_k \partial t} = 0. \quad (9)$$

Using the method proposed in ⁽⁶⁾, we integrate this equation over the thin transition layer between the bulk of the crystal and the vacuum, assuming that: 1) the tangential components of the electric field E_τ are continuous; 2) the electron mobility and the trap factor in the transition layer are equal to the corresponding values in the crystal; 3) the conductivity σ changes in the transition layer from the conductivity of the crystal to zero. After calculation we obtain:

$$\left[\frac{4\pi\sigma E_m}{k^2 f D - i\omega} - \frac{f D}{k^2 f D - i\omega} \frac{\partial}{\partial m} \text{div} \mathbf{D} + D_m \right]_{\text{cr}} = (D_m)_{\text{vac}}. \quad (10)$$

This boundary condition takes into account the conductivity and diffusion of electrons in the crystal. In the absence of electrons it reduces to the usual condition

$$(D_m)_{\text{cr}} = (D_m)_{\text{vac}}. \quad (11)$$

Fig. 1

Fig. 1

Figure 1: Fig. 1

If, in addition, there is a constant electric field in the crystal that creates an electron flux along the direction of propagation of the Rayleigh wave, then the expression $k^2 f D - i\omega$ in (10) is replaced by $k^2 f D - i\omega + ikE_0 f \mu$, where E_0 is the absolute magnitude of the drift field, $k = 2\pi/\lambda$ is the wave number.

We shall carry out the subsequent calculations assuming that we have a GaAs crystal bordering on vacuum along the plane $z = 0$ (Fig. 1), and that the Rayleigh wave propagates along the direction of diagonal 1 in this plane (the simplest direction along which a Rayleigh wave in a GaAs crystal is accompanied by a piezoelectric effect). We shall describe the Rayleigh wave by the displacement components $U_{x,y,z}$ and the electric potentials ψ and ψ_0 (in the crystal and in vacuum). From equations (1), (2), (9) we obtain the system

$$\rho \frac{\partial^2 U_i}{\partial t^2} = c_{iklm}^E \frac{\partial^2 U_m}{\partial x_k \partial x_l} + e_{jik} \frac{\partial^2 \psi}{\partial x_k \partial x_j},$$

$$e_{nlm} \frac{\partial^2 U_{lm}}{\partial t \partial x_n} - f D e_{nlm} \frac{\partial^3 U_{lm}}{\partial x_k^2 \partial x_n} - \sigma_0 \frac{\partial^2 \psi}{\partial x_k^2} \quad (12) - \frac{\varepsilon_{jn}^s}{4\pi} \frac{\partial^3 \psi}{\partial t \partial x_j \partial x_n} + \frac{f D \varepsilon_{jn}^s}{4\pi} \frac{\partial^4 \psi}{\partial x_k^2 \partial x_j \partial x_n} = 0.$$

The last equation has been linearized: in it we have neglected the term $\sim E_j^2$, which, for ordinary energy-flux densities in a Rayleigh wave (less than 1 W/cm²), is quite legitimate. We shall seek a solution of (12) in the form:

$$U_x = U_y = \frac{A}{k} \Phi(x, y, z, t), \quad U_z = \frac{B}{k} \Phi(x, y, z, t), \quad \psi = \frac{e}{k} C \Phi(x, y, z, t) \quad (13)$$

where

$$\Phi(x, y, z, t) = \exp \left[\beta k z + \frac{ik}{\sqrt{2}} (x + y) - i\omega t \right],$$

Here k is the as yet unknown wave number of the Rayleigh wave; β is a function of k ; e is the piezoelectric constant (in a GaAs crystal all nonzero components of the tensors of the piezoelectric constant and dielectric permittivity are equal to one another); A, B, C are dimensionless arbitrary constants. Substituting expressions (13) into equations (12), we obtain a system of 3 linear homogeneous algebraic equations with respect to A, B, C . Equating the determinant of this system to zero, we obtain an equation for finding the functions $\beta(k)$. It can be

shown that, from the entire set of roots of this equation, 3 roots correspond to the Rayleigh wave; it is expedient to write them in the form:

$$\beta_1 = \beta_1^0 + \delta_1, \quad \beta_2 = \beta_2^0 + \delta_2, \quad \beta_3 = 1 + \delta_3. \quad (14)$$

Here $\beta_{1,2}^0$ are the corresponding roots in the absence of a piezoelectric effect in the crystal ($e_{ijk} = 0$). After calculating $\beta_{1,2,3}$, two of the three arbitrary constants A, B, C can be expressed in terms of the third. As a result we obtain:

$$\begin{aligned} U_x = U_y &= \frac{1}{k} \sum_{n=1}^3 A_n \exp \left[\beta_n k z + \frac{ik}{\sqrt{2}}(x+y) - i\omega t \right], \\ U_z &= \frac{1}{k} \sum_{n=1}^3 F(\beta_n) A_n \exp \left[\beta_n k z + \frac{ik}{\sqrt{2}}(x+y) - i\omega t \right], \\ \psi &= \frac{e}{k} \sum_{n=1}^3 G(\beta_n) A_n \exp \left[\beta_n k z + \frac{ik}{\sqrt{2}}(x+y) - i\omega t \right]. \end{aligned} \quad (15)$$

Here $F(\beta_n), G(\beta_n)$ are functions of β_n and of the constants of the GaAs crystal; A_1, A_2, A_3 are new arbitrary constants.

For the electric potential ψ_0 in vacuum, from the equation $\Delta\psi_0 = 0$ and boundary condition (8) we obtain the expression $\psi_0 = \psi(0)e^{-kz}$, where $\psi(0)$ is the value of the potential in the crystal at $z = 0$. The remaining boundary conditions (7), (10) lead to the equations

$$\begin{aligned} \frac{\partial U_x}{\partial z} + \frac{\partial U_y}{\partial x} + \frac{e}{c_{44}^E} \frac{\partial \psi}{\partial y} &= 0, \quad \frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} + \frac{c_{11}^E}{c_{12}^E} \frac{\partial U_z}{\partial z} = 0, \\ e \left(\frac{\partial U_x}{\partial y} + \frac{\partial U_y}{\partial x} \right) + \frac{1}{4\pi} \frac{\partial \psi}{\partial z} - \frac{\varepsilon^s}{4\pi} \frac{\partial \psi}{\partial z} - \frac{i\sigma}{\omega} \frac{\partial \psi}{\partial z} - \frac{ifD}{\omega} \left(4e \frac{\partial^3 U_x}{\partial y \partial z^2} + 2e \frac{\partial^3 U_z}{\partial x \partial y \partial z} - \varepsilon^s \Delta \frac{\partial \psi}{\partial z} \right) &= 0, \end{aligned} \quad (16)$$

where the values of all functions and of their derivatives are taken at $z = 0$. Substituting expressions (15) into equations (16) and setting to zero the determinant of the resulting system of linear homogeneous equations, we obtain the dispersion equation for finding the wave number k .

This equation is very cumbersome. We shall seek its solution by the method of successive approximations, putting $k = k_0(1 + \alpha)$, where k_0 is the wave number of a Rayleigh wave in the crystal in the absence of the piezoelectric effect. Using expressions (14) for β_i and expanding all quantities in powers of the small parameter e^2/c_{44}^E (its value for GaAs is $3.647 \cdot 10^{-3}$), we obtain, in

Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

the zeroth approximation, an equation for k_0 . The next approximation gives an equation for α , which after a number of calculations can be written in the form

$$\left(\frac{3.379\omega_c/\omega}{1 + i\omega/\omega_D} - 3.256i \right) \alpha = 0.393 \frac{e^2}{c_{44}^E} i++ \left[0.658 - 0.0443i + \frac{(0.032 + 0.716i)\omega_c/\omega}{1 + i\omega/\omega_D} \right] \delta_1++ \left[-0.658 - 0.0443i + \right.$$

Here the notation is $\omega_c = \sigma_0$; $\omega_D = c_0^2/fD$; $\delta'_{1,2} = \delta_{1,2}$ for $\sigma = 0$,

$$\delta_1 = \frac{(-1.05 + 0.327i)e^2/c_{44}^E}{1 + \frac{\omega_c/\omega}{(0.865 - 0.425i)\omega/\omega_D - 0.883i}}, \quad \delta_2 = \frac{(-1.05 - 0.327i)e^2/c_{44}^E}{1 + \frac{\omega_c/\omega}{(0.865 + 0.425i)\omega/\omega_D - 0.883i}}.$$

In deriving this equation we used the values of the constants of the GaAs crystal taken from Refs. (7,8).

Figures 2 and 3 show the resulting curves for the interaction of Rayleigh waves with electrons in a GaAs crystal. Along the abscissa axes, on a logarithmic scale, is plotted the ratio of the crystal conductivity to the frequency; along the ordinate axes are plotted the relative change $\Delta c/c_0 = -\text{Re } \alpha$ of the phase velocity of the Rayleigh wave (curves 1) and the attenuation coefficient, referred to 2π , $\gamma = \text{Im } \alpha$, of the Rayleigh wave per wavelength (curves 2). Figure 2 refers to the case $\omega_D = \infty$ (there is no electron diffusion in the crystal), and Fig. 3 to the case $\omega_D = 0.1\omega_c$. As is seen from the figures, to each value of the crystal conductivity ω_c there corresponds a frequency at which the interaction of the Rayleigh wave with the electrons is maximal: the attenuation γ and the change in velocity are maximal.

Fig. 2

Fig. 3

With increasing electron diffusion (decreasing ω_D) the interaction weakens. The character of these dependences is analogous to that of the corresponding curves for transverse waves (1) propagating in the GaAs crystal in the same direction 1. Quantitatively, however, the curves differ greatly. Rayleigh waves interact with electrons substantially more weakly than transverse waves: their attenuation γ and the change in phase velocity are approximately a factor of 2 smaller

than the corresponding quantities for transverse waves. A probable cause of this difference may be the demagnetizing effect ⁽⁹⁾, manifested for the Rayleigh wave because of its localization in a thin surface layer. Electron diffusion has a very strong influence on the interaction of a Rayleigh wave with electrons in GaAs. As is seen from comparison of Figs. 2 and 3, it eliminates the interaction for $\omega > \omega_c$ and greatly reduces it at all other frequencies, including in the region $\omega \ll \omega_c$. For bulk waves the latter is not observed. The enhanced role (compared with the case of bulk waves) of electron diffusion in the interaction of a Rayleigh wave with the crystal's conduction electrons is apparently caused by the addition, in the Rayleigh-wave problem, of boundary conditions that contain the diffusion coefficient, as well as by the inhomogeneity of the Rayleigh wave with depth (owing to which electron diffusion occurs both along the direction of wave propagation and along the z axis).

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Note: Figure translations are in progress. See original paper for figures.

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