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Abstract

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MATHEMATICS

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ON TWO SUBCLASSES OF TURING MACHINES REDUCIBLE TO FINITE AUTOMATA

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Two subclasses of Turing machines are considered—two-way automata and machines operating in a restricted mode. As is known, the truth sets of predicates computable on such Turing machines are sets representable in finite automata (¹⁻³). In the present note an estimate is given for the number of states of the corresponding finite automata.

Fix an input alphabet Σ of a machine. Let $\mathfrak{T}_n$ be the class of machines T with input alphabet Σ , having no more than n states and such that the truth set of the recursively enumerable predicates computable by them is representable in a finite automaton. Let A_T be the reduced automaton corresponding to the machine T , and let $|A_T|$ be the number of its states. Define the function $\Phi(n)$:

$$\Phi(n) = \max_{T \in \mathfrak{T}_n} |A_T|. \quad (1)$$

By the methods used in (⁴), the following can easily be obtained.

Theorem 1. *For any general recursive function $f(n)$ there exists a number n_0 such that for all $n > n_0$, $f(n) < \Phi(n)$. Thus, the function $\Phi(n)$ is not general recursive.*

We introduce two subclasses of Turing machines.

We shall consider Turing machines with working alphabet $\Xi \supseteq \Sigma$ and set of states Q , whose operation is determined by the functions

$$\varphi_1 : Q \times \Xi \rightarrow Q, \quad \varphi_2 : Q \times \Xi \rightarrow \Xi, \quad \varphi_3 : Q \rightarrow \{R, L\}.$$

The machine, being in state $q \in Q$ and reading from the tape the symbol $\xi \in \Xi$, prints the symbol $\xi' = \varphi_2(q, \xi)$, passes to the state $q' = \varphi_1(q, \xi)$, and moves one step to the right if $\varphi_3(q') = R$, or one step to the left if $\varphi_3(q') = L$. Thus, the motion of the machine is determined only by the state into which the machine enters. The function φ_3 partitions Q into two subsets $Q = Q^R \cup Q^L$ such that

$Q^R = \{q : \varphi_3(q) = R\}$, $Q^L = \{q : \varphi_3(q) = L\}$. Let $|Q^R| = r$, $|Q^L| = l$. Further, let the initial state of the machine q_0 be such that $q_0 \in Q^R$. The machine computes a predicate in the following sense: before the start of operation, a word P in the alphabet Σ is written on the tape, to which the machine is applied. The machine stops if and only if its head leaves the portion of the tape initially occupied by the word P ; moreover, if the head leaves the tape portion on the right, and the machine is in a state from some subset $F \subseteq Q^R$, then the computed predicate is true on the word P .

The first subclass we consider is two-way automata ^(1,2). These are Turing machines for which, for all $q \in Q$ and $\xi \in \Sigma$, $\varphi_2(q, \xi) = \xi$, i.e., they are nonwriting and nonerasing machines. By analogy with (1), introduce the function $\Phi_1(r, l)$.

Theorem 2. If $|\Sigma| \geq 3$, $l \geq 1$, then $\Phi_1(r, l) = r^{l+1} + 1$.

Suppose that the word P is written on the tape. Fix a boundary between two neighboring cells. While working on the word P , the machine crosses this boundary for the first time in a state $q^R \in Q^R$, for the second time in a state $q^L \in Q^L$, and so on. The word thus obtained is the trace at the given boundary (or at the given point).

The second subclass we consider is Turing machines operating in a bounded mode ⁽³⁾, i.e., machines for which there exists a number c such that, if the machine halts on the word P , then the length of the trace at any point of the tape is bounded by c . By analogy with (1), define the function $\Phi_2(r, l, c)$.

Theorem 3. For $|\Sigma| \geq 2$,

$$\left(r - \left\lfloor \frac{c-1}{2} \right\rfloor - 3\right)^{l \left\lceil \frac{c-1}{2} \right\rceil} \ll \Phi_2(r, l, c) \ll (r+1)^{\frac{l \left\lceil \frac{c+1}{2} \right\rceil}{l-1}}.$$

Let $\mathfrak{S}_n$ be the class of Turing machines operating in a bounded mode and having no more than n states. For $T \in \mathfrak{S}_n$, denote by c_T the constant bounding the lengths of the traces of the machine T , and define the function $\Phi_3(n)$:

$$\Phi_3(n) = \max_{T \in \mathfrak{S}_n} c_T;$$

using the method of ⁽⁴⁾, it is easy to obtain that Theorem 1 holds if in it, as $\Phi(n)$, one takes the function $\Phi_3(n)$.

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³ B. A. Trakhtenbrot, *Algebra i logika*, Seminar, 3, 1964, p. 33.

⁴ Rado, *Bell System Technology J.*, 41, No. 3, 877 (1962).

Note: Figure translations are in progress. See original paper for figures.

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