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Abstract

Full Text

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MATHEMATICS

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SOLUTION OF DISCRETE PROGRAMMING PROBLEMS BY THE METHOD OF CONSTRUCTING A SEQUENCE OF PLANS

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Consider problem A of minimizing the functional

$$F(X) = g_1(x_1)\omega g_2(x_2)\omega \dots \omega g_m(x_m)\omega Q(x_1, x_2, \dots, x_m)$$

subject to the constraints

$$\sum_{i=1}^m a_i x_i \leq c; \quad (1)$$

$$X = (x_1, x_2, \dots, x_m) \in G, \quad (2)$$

$$x_i \in \gamma_i = \{a_{i1}, a_{i2}, \dots, a_{is_i}\}, \quad i = 1, \dots, m, \quad (3)$$

where a_i, c, a_{ij} are positive integers; ω is the operation of addition or multiplication; $Q(X)$ is a function for which there exist functions $t_i(x_i)$, $i = 1, \dots, m$, such that

$$t_1(x_1)\omega t_2(x_2)\omega \dots \omega t_m(x_m) \leq Q(X)$$

for any plan X of problem A. The set G may be specified in various ways (by equations, inequalities, logical conditions, etc.). Denote the set of plans of problem A by M , and the set of plans determined by constraints (1) and (3) by $\overline{M}$.

In the present note we propose a method for solving problem $\overline{A}$, consisting in the construction and analysis of a sequence of plans of problem $\overline{A}$ for minimizing the functional $\overline{F}(X) = \overline{g}_1(x_1)\omega \overline{g}_2(x_2)\omega \dots \omega \overline{g}_m(x_m)$ on the set $\overline{M}$. Here $\overline{g}_i(x_i) = g_i(x_i)\omega t_i(x_i)$, $i = 1, \dots, m$.

Let $f_h(z)$ be the optimal value of the functional of problem $\bar{A}$, in which the index m and the parameter c are replaced respectively by h and z ⁽¹⁾. A plan $X_0 = (x_1^0, x_2^0, \dots, x_m^0)$ of problem $\bar{A}$ will be called h -optimal ($1 \leq h \leq m$) and denoted by

$$(*, *, \dots, *, x_{h+1}^0, \dots, x_m^0),$$

if

$$\bar{F}(X_0) = \bar{g}_{h+1}(x_{h+1}^0) \omega \dots \omega \bar{g}_m(x_m^0) \omega f_h \left(c - \sum_{i=h+1}^m a_i x_i^0 \right).$$

In this case the set of plans of problem $\bar{A}$ such that $x_i = x_i^0$, $i = h+1, \dots, m$, will be denoted by

$$\bar{M}(*, *, \dots, *, x_{h+1}^0, \dots, x_m^0) = \bar{M}(X_0).$$

Directly from the definition of h -optimality of the plan X_0 it follows that

Lemma 1. $\bar{F}(X_0) = \min_{X \in \bar{M}(X_0)} \bar{F}(X)$.

We describe an algorithm φ for constructing a sequence of plans of problem A .

First step. Among the set W_1 of all $(m-1)$ -optimal plans of problem $\bar{A}$ we find such a plan

$$X_1 = (*, *, \dots, *, x_m^1) = (x_1^1, x_2^1, \dots, x_m^1),$$

such that

$$\bar{F}(X_1) = \min_{X \in W_1} \bar{F}(X).$$

k -th step ($k = 2, 3, \dots$). We transform the set W_{k-1} into the set W_k according to the following rule. We leave unchanged all plans of the set W_{k-1} , except for the plan

$$X_{k-1} = (*, *, \dots, *, x_p^{k-1}, \dots, x_m^{k-1}) = (x_1^{k-1}, x_2^{k-1}, \dots, x_m^{k-1}),$$

which we replace by the system of all possible plans of the form

$$(*, *, \dots, *, x_{q-1}^{k-1}, x_q^{k-1}, \dots, x_p^{k-1}, \dots, x_m^{k-1}),$$

where $x_{q-1} \in Y_{q-1}$, $x_{q-1} \neq x_{q-1}^{k-1}$, $q = 2, 3, \dots, p$, and

$$\sum_{i=1}^{q-2} \max_{1 \leq k \leq s_i} a_{ik} \geq c - \sum_{i=q}^m x_i^{k-1} - x_{q-1}.$$

Next we find such a plan X_k that

$$\bar{F}(X_k) = \min_{X \in W_k} \bar{F}(X),$$

and pass to the next step.

Introduce the notation

$$\bar{M}_k = \bigcup_{X \in W_k} \bar{M}(X).$$

Lemma 2. $\bar{M}_1 = \bar{M}$, $\bar{M}_k = \bar{M}_{k-1} \setminus X_{k-1}$, $k = 2, 3, \dots$

From Lemmas 1 and 2 it follows that

Theorem 1. The algorithm constructs such a sequence of plans $X_1, X_2, \dots$ of problem $\bar{A}$ that

$$\bar{F}(X_k) = \min_{X \in \bar{M} \setminus \{X_1, X_2, \dots, X_{k-1}\}} \bar{F}(X), \quad k = 1, 2, \dots$$

If, from the constructed sequence of plans $X_1, X_2, \dots$ of problem $\bar{A}$, we remove the plans that do not belong to the set G , then for the remaining sequence of plans $X'_1, X'_2, \dots$ of problem A the following holds.

Theorem 2. If there exists a number k such that

$$F(X'_k) \geq \min\{F(X'_1), F(X'_2), \dots, F(X'_{k-1})\} = F(X^*),$$

then X^* is an optimal plan of problem A .

Theorem 2 specifies an algorithm ψ for solving problem A . The desired algorithm ψ is described as follows: the k -th step consists in constructing, with the aid of algorithm φ , the plan X_k and checking whether it belongs to the set G . If not, we pass to the next step of algorithm ψ . If yes, we check whether the condition of Theorem 2 is satisfied. If not, we pass to the next step. If yes, the process terminates and X^* is an optimal plan.

The method presented is applicable to solving facility location problems ⁽²⁻⁴⁾, integer programming problems ^(5, 6), reliability problems for a complex system ⁽⁷⁾, and the traveling-salesman problem.

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Note: Figure translations are in progress. See original paper for figures.

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