

ON AN INVERSE PROBLEM FOR THE TELEGRAPH EQUATION

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Abstract

Full Text

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MATHEMATICS

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ON AN INVERSE PROBLEM FOR THE TELEGRAPH EQUATION

(Presented by Academician G. I. Marchuk, 24 IV 1969)

This paper is devoted to finding the coefficient for the telegraph equation.

Consider the equation

$$u_{xy} = au \quad (1)$$

with data on the characteristics

$$u(x, y, \lambda)|_{x=0} = g(y - \lambda), \quad u(x, y, \lambda)|_{y=0} = 0, \quad (2)$$

$$u(x, y, \lambda)|_{y=c} = \varphi(x, \lambda), \quad (3)$$

where $u(x, y, \lambda)$ is a family of solutions of (1), depending on the parameter λ ($0 < \lambda < c$); $a = a(x, y)$ is the coefficient of equation (1); $g(y - \lambda)$ is the Dirac function; $\varphi(x, \lambda)$ is a function having a continuous mixed second derivative with respect to x and λ in the rectangle $Q = [0, h; 0, c]$. The rectangle Q is bounded by the coordinate axes and the straight lines $x = h$, $y = c$; h and c are positive constants.

Let $a(x, y)$ be an unknown continuous function in the rectangle Q , and let

$$\sup_{0 \leq x \leq h, 0 \leq y \leq c} |a(x, y)| = M,$$

where M is some positive number.

Let c and h satisfy the condition

$$M^* \operatorname{ch} S < 1, \quad (4)$$

where S and M^* are certain positive numbers, which will be discussed below.

Multidimensional inverse problems in a linearized formulation were considered in work (1).

In this paper the problem is considered in the following formulation: for a given function $\varphi(x, \lambda)$ in the rectangle Q , it is required to find the coefficient $a(x, y)$ of equation (1) in this rectangle.

The solution of equation (1) satisfying the boundary conditions (2) is equivalent to the solution of the integral equation:

$$u(x, y, \lambda) = g(y - \lambda) + \int_0^y \int_0^x d(\xi, \eta) u(\xi, \eta, \lambda) d\xi d\eta. \quad (5)$$

Equation (5) is a Volterra integral equation of the second kind with respect to $u(x, y, \lambda)$.

From (5), applying the method of successive approximations, we obtain

$$u(x, y, \lambda) = g(y - \lambda) + \sum_{n=1}^{\infty} \int_0^y \int_0^x g(t - \lambda) a_n(x, y, \tau, t) d\tau dt, \quad (6)$$

where

$$a_n(x, y, \tau, t) = \int_t^y \int_\tau^x a(\xi, \eta) a_{n-1}(\xi, \eta, \tau, t) d\xi d\eta$$

is the n -th iterated kernel for $a(x, y)$. In view of the boundedness of $a(x, y)$ in the rectangle Q , the series in (6) will converge absolutely and uniformly in this rectangle. Therefore, interchanging the order of summation and integration in (6), we obtain

$$u(x, y, \lambda) = g(y - \lambda) + \int_0^y \int_0^x g(t - \lambda) R(x, y, \tau, t) d\tau dt, \quad (7)$$

where

$$R(x, y, \tau, t) = \sum_{n=1}^{\infty} a_n(x, y, \tau, t), \quad a_1 = a(\tau, t).$$

From (7), by virtue of condition (3), we obtain

$$\varphi(x, \lambda) = g(c - \lambda) + \int_0^c \int_0^x g(t - \lambda) R(x, c, \tau, t) d\tau dt. \quad (8)$$

Differentiating relation (8) with respect to x , and then with respect to λ , we obtain

$$-\varphi''_{x\lambda}(x, \lambda) = a(x, \lambda) + \int_{\lambda}^c \int_0^x a(x, \eta) R(x, \eta, \tau, \lambda) d\tau d\eta. \quad (9)$$

Relation (9) can be written in the form

$$a(x, \lambda) = Aa, \quad (10)$$

where

$$Aa = -\varphi''_{x\lambda}(x, \lambda) - \int_{\lambda}^c \int_0^x a(x, \eta) R(x, \eta, \tau, \lambda) d\tau d\eta \quad (11)$$

is a nonlinear operator.

We now show that this operator is a contraction operator in the space $C[0, h; 0, c]$. Take two arbitrary elements $a(x, y)$ and $b(x, y)$ from the space $C[0, h; 0, c]$,

$$\rho(a, b) = \max_{x, y \in C[0, h; 0, c]} |a - b|.$$

Let for $b(x, y)$ in the rectangle Q the condition

$$\sup_{0 \leq x \leq h, 0 \leq y \leq c} |b(x, y)| = M_1$$

be satisfied.

Denote by M^* the larger of the numbers M and M_1 . Using relation (11), we write $\rho(Aa, Ab)$:

$$\begin{aligned} \rho(Aa, Ab) &= \|Aa - Ab\| = \\ &= \left\| \int_{\lambda}^c \int_0^x [b(x, \eta) R^*(x, \eta, \tau, \lambda) - a(x, \eta) R(x, \eta, \tau, \lambda)] d\tau d\eta \right\|, \end{aligned} \quad (12)$$

where

$$R^*(x, \eta, \tau, \lambda) = \sum_{n=1}^{\infty} b_n(x, \eta, \tau, \lambda), \quad b_1 = b(\tau, \lambda).$$

From (12) we obtain

$$\rho(Aa, Ab) \leq \rho(a, b)M^*hcS, \quad (13)$$

where S is the sum of the series

$$\sum_{n=1}^{\infty} \frac{(n+1)|M^*hc|^{n-1}}{(n-1)!(n-1)!}.$$

From (13), in view of condition (4), we see that the operator A is a contraction operator. Therefore, on the basis of the contraction mapping principle, from the functional equation (10) we determine the unique coefficient $a(x, y)$ in the rectangle Q .

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CITED LITERATURE

1. M. M. Lavrent'ev, V. G. Romanov, DAN, **171**, No. 6 (1966).
2. S. Elubaev, Vestn. AN KazSSR, 1969.

Note: Figure translations are in progress. See original paper for figures.

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