

A Nonparametric Method for Determining the Depth of the Top of the Crystalline Basement from Frequency-Sounding Curves

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1968

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

UDC 550.837

GEOPHYSICS

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A Nonparametric Method for Determining the Depth of the Top of the Crystalline Basement from Frequency-Sounding Curves

(Presented by Academician A. V. Peive on 10 VIII 1967)

One of the principal geological problems usually posed for the method of frequency soundings (FS) is the determination of the depth and relief of the top of the crystalline basement. The FS curves obtained under these conditions reflect various multilayer geoelectric structures with a marker horizon $\mu_n \rightarrow \infty$, and their right-hand branches can correspond to only two types of three-layer curves: H or A .

We shall explain the essence of the proposed method using as an example the amplitude curves for the vertical component of the magnetic field (B_z). Let us first turn to two-layer curves of type $\mu_2 \rightarrow \infty$. Analysis of these curves, both for the magnetic and for the electric components of the field, makes it possible to establish a number of relationships between the parameters of the two-layer section, the spacing with which the sounding was obtained, and the characteristic points of the curve.

Curve *I* in Fig. 1 depicts the relationship between the amplitudes A of the right-hand branches of the FS curves for the component B_z and the quantities r/H_1 ; r is the spacing. H_1 is the thickness of the upper layer. Curve *II* characterizes the dependence of the abscissae λ_1/H_1 of the maxima of the FS curves for the component B_z on r/H_1 ; λ_1 is the wavelength in the upper layer. Curve *III* characterizes the dependence of the abscissae λ_1/H_1 of the minima of the curves on r/H_1 .

Fig. 1

It follows from Fig. 1 that, in the case of a two-layer section of type $\mu_2 \rightarrow \infty$, determining the thickness of the upper layer is extremely simple: the amplitude of the right-hand branch of the FS curve obtained (from the axis $\rho = \rho_1$ to its

maximum) is measured in centimeters, and from curve I the value of r/H_1 is found. Since r is known, H_1 becomes known.

The dependence between the quantities A , $(\lambda_1/H_1)_{\max}$, and $(\lambda_1/H_1)_{\min}$ on r/H_1 can be represented analytically with an error of up to 1%.

Between A and r/H_1 there is the dependence

$$A = 1.71 (r/H_1 - 1.7)^{0.63}, \quad (1)$$

whence

$$H_1 = 2.33r / (A^{1.59} + 3.96). \quad (2)$$

Taking into account that FS curves are plotted on a double logarithmic scale and taking the construction modulus equal to 1 : 10, we find

$$A = 10(\lg \rho_m - \lg \rho_1) = 10 \lg(\rho_m / \rho_1), \quad (3)$$

where ρ_m is the value of ρ at the maximum of the right branch of the curve, and ρ_1 is the longitudinal resistivity of the upper layer.

From (2) and (3) we obtain

$$H_1 = 2.33r / [(10 \lg \rho_m / \rho_1)^{1.59} + 3.96]. \quad (4)$$

Between $(\lambda_1/H_1)_{\max}$ and r/H_1 there is the dependence

$$(\lambda_1/H_1)_{\max} = 2.95(r/H_1)^{0.6}. \quad (5)$$

Taking into account that

$$(\lambda_1/H_1)_{\max} = \frac{3.16\sqrt{\rho_1}}{H} \left(\frac{1}{\sqrt{f}} \right)_{\max},$$

we find

$$H_1 = 1.18\rho_1^{1.25} (1/\sqrt{f})_{\max}^{2.5} / r^{1.5}. \quad (6)$$

Between $(\lambda_1/H_1)_{\min}$ and r/H_1 there is the dependence

$$(\lambda_1/H_1)_{\min} = 2.1(r/H_1)^{0.4}. \quad (7)$$

Since

$$(\lambda_1/H_1)_{\min} = \frac{316\sqrt{\rho_1}}{H_1} \left(\frac{1}{\sqrt{f}} \right)_{\min},$$

then

$$H_1 = 1.97\rho_1^{0.83}(1/\sqrt{f})_{\min}^{1.67}/r^{0.67}. \quad (8)$$

The quantity H_1 is related to the longitudinal conductance (S_1) and ρ_1 by the known relation

$$H_1 = \rho_1 S_1. \quad (9)$$

For two-layer and multilayer theoretical and experimental FS curves, the quantities S_1 , $(1/\sqrt{f})_{\max}$, and $(1/\sqrt{f})_{\min}$ are determined from the curve itself, and the value of r is known; therefore, the correct determination of H_1 depends on the correct choice of ρ_1 .

Each of formulas (4), (6), (8), and (9) characterizes different laws of variation of H_1 as a function of the variation of ρ_1 . They give identical values of H_1 only when ρ_1 is chosen to correspond to the true value; otherwise, the resulting H_1 are different.

Any multilayer FS curve reflecting a section with a reference horizon having $\mu_n \rightarrow \infty$ can be represented by an equivalent two-layer curve in which ρ_1 is equal to the average longitudinal resistivity of all layers of the section (ρ_l), and H_1 is equal to the total thickness (H) of all layers of the section down to the reference horizon. Therefore, everything said concerning two-layer curves applies equally to multilayer curves in which the lower layer has $\mu_n \rightarrow \infty$.

This makes it possible to interpret multilayer FS curves of the type $\mu_n \rightarrow \infty$ on the basis of the obtained formulas (4), (6), (8), and (9) by testing different values of ρ_l . In this case, the true value of H will be the one obtained identically by all formulas. At the same time, the true value of ρ_l is also determined.

It should be noted that each of formulas (4), (6), (8), and (9) has a different resolving power, expressed in the fact that a change in ρ_l by a given amount leads to different changes in the value of H . On average, when ρ_l is changed by 100%, the discrepancies between the values of H obtained by the different formulas reach 25–30%.

It appears possible, however, to increase the resolving power of the method and to bring it on average to 100% instead of 25–30%. This is achieved by various combinations among the obtained formulas.

It proves advantageous to use formulas (10) and (11), of which the first was obtained by combining (4), (8), and (9), while the second is the result of combining (8) and (9):

Fig. 2

Figure 2: Fig. 2

$$H = 4,59 r^{0,33} (1/\sqrt{f})_{\min}^{1,6} / [(10 \lg \rho_m / \rho_l)^{1,59} + 3,96] \rho_l^{0,17} S; \quad (10)$$

$$H = 0,51 S^2 \rho_l^{1,17} r^{0,67} / (1/\sqrt{f})_{\min}^{1,67}. \quad (11)$$

It follows from formulas (10) and (11) that, in order to carry out the interpretation, it is necessary to use the known value of r and to determine from the obtained frequency-sounding curve the quantities S , ρ_m , and $(1/\sqrt{f})_{\min}$; moreover, if the first three of these are constant for the curve, the fourth will vary depending on the value of ρ_l chosen for interpretation.

The value $(1/\sqrt{f})_{\min}$ is determined as follows. The horizontal axis $\rho = 1$ of the two-layer palette $\mu_2 = \infty$ is superposed with the value of ρ_l chosen for interpretation, and one notes with which palette curve the maximum of the curve being interpreted completely coincided. The sought value $(1/\sqrt{f})_{\min}$ is the abscissa of the FS curve with which the abscissa of the minimum of the palette curve coincided.

Fig. 2

Let us turn to several illustrations. In Fig. 2, I represents a three-layer FS curve for the component B_z with values $\mu_2 = 1/4$ and $\nu_2 = 1$, $\mu_3 = \infty$ ($r = 10$ km). It can be represented by an equivalent two-layer curve for which $\rho_1 = 0,4 \Omega \cdot \text{m}$ and $H_1 = 2$ km.

Suppose, as is the case in reality, that the parameters of the section characterized by this curve are unknown to us, and only the sounding length r is known. We determine from the curve the value S by the known method (1) ($S = 5$) and carry out its interpretation by successive trials of various values of ρ_l , determining from formulas (10) and (11) the corresponding values of H . The data obtained are collected in Table 1, from which it is seen that the values of H are obtained correctly and coincide with one another by both formulas (1.92 km instead of 2 km) when ρ_l is close to the true value ($0,4 \Omega \cdot \text{m}$). For other values of ρ_l , the values of H do not coincide with one another, and none of them corresponds to reality.

Table 1

	0,26	0,29	0,33	0,47	0,58	0,58
ρ_l	0,26	0,29	0,33	0,47	0,58	0,58
H (10)	1,17	1,33	1,62	2,07	2,73	3,63
H (11)	1,38	1,54	1,71	1,95	2,3	2,82

	0,26	0,29	0,33	0,47	0,58	0,58
Discrepancy between (10) and (11), in %	18	16	6	3	19	29

In Fig. 2 the curve *II* characterizes a four-layer section of type *KH* ($\mu_2 = \infty$, $\nu_2 = 1/4$, $\mu_3 = 1/4$, $\nu_3 = 2$, $\mu_4 = \infty$), $r = 16$ km. Table 2 contains the results of the interpretation. The obtained values for $H = 3,25$ and $3,3$ km ($\rho_l = 0,35 \Omega \cdot \text{m}$) agree well with their actual values, equal to 3.25 km.

Table 2

ρ_l	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6
H	1.25	1.9	2.58	3.25	3.85	4.65	5.48	6.4	7.2
(10)									
H	2.48	2.7	2.95	3.3	3.65	4.1	4.45	4.8	5.15
(11)									
Discrepancy between (10) and (11), %	100	42	14	1.5	5.5	13	22	35	40

It is important that the proposed procedure makes it possible to carry out a parameter-free interpretation of multilayer frequency-sounding curves ending in a three-layer part of type *A*, which is not accessible to existing methods. This assertion requires no special proof, since on the right branch of a three-layer curve of type *A* one can find all the required quantities (ρ_m , $(1/\sqrt{f})_{\max}$, $(1/\sqrt{f})_{\min}$, and S) needed to determine the quantities H and ρ_l . At the same time, it is naturally necessary that the resistivity of the overlying bed not be equal or close to the resistivity of the reference horizon.

Nevertheless, an extensive check was carried out, fully confirming the possibility of parameter-free interpretation of curves of this type.

As an illustration, Fig. 2 III shows a three-layer curve of type *A* ($\rho_1 = 1 \text{ ohm} \cdot \text{m}$, $H_1 = 1 \text{ km}$, $\mu_2 = 2$, $\nu_2 = 2$, $\mu_3 = \infty$, $\nu_3 = \infty$, $r = 16 \text{ km}$).

Table 3

ρ_l	0.8	1	1.2	1.5	1.6	1.8	2.3
H	1.3	1.78	2.34	2.93	3.44	4.5	6.1
(10)							

ρ_l	0.8	1	1.2	1.5	1.6	1.8	2.3
H	2.00	2.35	2.7	3.05	3.11	3.45	4.21
(11)							
Discrepancy	54	32	15	4	11	31	47
be-							
tween							
(10)							
and							
(11),							
%							

Table 3 gives the results of the interpretation. The values $H = 2.93$ and 3.05 km ($\rho_l = 1.5$ ohm $\cdot$ m), obtained close to one another, correspond to their true values.

From Tables 1, 2, and 3 one can see the resolving power of the method when formulas (10) and (11) are used.

To determine the limits of applicability of the described method, it is necessary to carry out additional calculations of theoretical FS curves for comparison with those already available.

However, in general it appears clear that the method will fail to give satisfactory accuracy only for those groups of geoelectric structures for which, for a given spacing (taking account of measurement error), the coordinates of the right-hand branches of the FS curves (including the maximum) turn out to be identical.

In conclusion, we note that, besides the combinations (10) and (11) given in the article, other combinations of the initial formulas ((4), (6), (8), and (9)) are also possible; depending on the geoelectric parameters of the section, in the interpretation of the sounding curves obtained they may yield results no less satisfactory.

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Received
8 VIII 1967

CITED LITERATURE

1. B. S. Epshtein, *Izv. AN SSSR*, geophys. ser., No. 12 (1957).

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