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Abstract

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PHYSICS

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ELECTROMAGNETIC PROPERTIES OF SUPERCONDUCTORS WITH STRONG COUPLING

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The usual theory of superconductivity, as is well known, is developed in the weak-coupling approximation. There are, however, a number of superconducting substances (for example Pb, Hg, Ga, NbN, Bi films) in which the electron-phonon interaction is not weak and whose properties are poorly described by this usual theory (for a review of experimental data see, for example, ⁽¹⁾). In the authors' papers ^(2, 3) the question of the energy gap in superconductors with strong coupling is investigated, and their thermodynamic and kinetic properties are considered. Below we investigate the electromagnetic properties of anomalous superconductors. Their behavior in an arbitrary magnetic field near T_k is considered, the Ginzburg-Landau equations are obtained, the penetration depth of a weak field is calculated, and the electromagnetic properties of alloys in the presence of strong coupling are investigated.

1. Strong magnetic fields. The Ginzburg-Landau equations. We write the equations for the thermodynamic Green functions $G_\omega(\mathbf{r}, \mathbf{r}')$ and $F_\omega^+(\mathbf{r}, \mathbf{r}')$ in the presence of a magnetic field:

$$\left\{ i\omega + \frac{1}{2m}(\nabla_r - ie\mathbf{A})^2 + \mu \right\} G_\omega(\mathbf{r}, \mathbf{r}') - \int d\mathbf{r}'' F_\omega^+(\mathbf{r}'', \mathbf{r}') \Sigma_\omega(\mathbf{r}, \mathbf{r}'') = \delta(\mathbf{r} - \mathbf{r}'),$$

$$\left\{ -i\omega + \frac{1}{2m}(\nabla_r + ie\mathbf{A})^2 + \mu \right\} F_\omega^+(\mathbf{r}, \mathbf{r}') - \int d\mathbf{r}'' G_\omega(\mathbf{r}'', \mathbf{r}') \tilde{\Sigma}_\omega(\mathbf{r}, \mathbf{r}'') = 0; \quad (1)$$

$$\Sigma_\omega(\mathbf{r}, \mathbf{r}'') = g^2 T \sum_{\omega'} F_{\omega'}(\mathbf{r}, \mathbf{r}'') D_{\omega-\omega'}(\mathbf{r} - \mathbf{r}'). \quad (2)$$

Introducing, analogously to ⁽⁴⁾, the Green function

$$\widetilde{G}^{(0)}(\mathbf{r}, \mathbf{r}') = G_{\omega}^{(0)}(R) e^{ie\mathbf{A} \cdot (\mathbf{r} - \mathbf{r}')} ; \quad G_{\pm\omega}^{(0)} = -\frac{m}{2\pi R} e^{\pm ip_0 R - \frac{|\omega|}{v_F} R}$$

for the normal metal and then writing equations (1), (2) in integral form, after expanding at $T \rightarrow T_k$ we arrive at the following equation for the proper-energy part $\Sigma_{\omega}(\mathbf{r}, \mathbf{r}')$ describing Cooper pairing:

$$\begin{aligned} \Sigma_{\omega}(\mathbf{r}, \mathbf{r}') = g^2 T \sum_{\omega'} \int d\mathbf{r}'' d\mathbf{l} \widetilde{G}_{-\omega'}^{(0)}(\mathbf{l}, \mathbf{r}) \widetilde{G}_{\omega'}^{(0)}(\mathbf{r}'', \mathbf{r}') \widetilde{\Sigma}_{\omega}(\mathbf{l}, \mathbf{r}'') D_{\omega-\omega'}(\mathbf{r} - \mathbf{r}') - g^2 T \sum_{\omega'} \int \widetilde{G}_{-\omega'}^{(0)}(\mathbf{l}, \mathbf{r}) \widetilde{G}_{\omega'}^{(0)}(\mathbf{r}'', \mathbf{l}_2) \widetilde{G}_{-\omega'}^{(0)}(\mathbf{l}_2, \mathbf{r}') \\ \times \widetilde{\Sigma}_{\omega}(\mathbf{l}_1, \mathbf{r}') \Sigma_{\omega}(\mathbf{l}_2, \mathbf{r}_2) \widetilde{\Sigma}_{\omega}(\mathbf{l}, \mathbf{r}'') D_{\omega-\omega'}(\mathbf{r} - \mathbf{r}') d\mathbf{r}_1 d\mathbf{l}_1 d\mathbf{r}_2 d\mathbf{l}_2 d\mathbf{r}'' d\mathbf{l}. \end{aligned}$$

The essential distances determining the region in which the values of the phonon function $D_{\omega-\omega'}(\mathbf{r} - \mathbf{r}')$ are noticeably different from zero are of the order of interatomic distances. The functions $G_{\omega}^{(0)}(\mathbf{r}, \mathbf{r}')$, however, decrease appreciably only at distances greater than ξ_0 ⁽⁴⁾. Therefore $D_{\omega-\omega'}(\mathbf{r} - \mathbf{r}')$ may be regarded as proportional to a δ -function, and the corresponding integration may be performed. The function $\Sigma_{\omega}(\mathbf{r})$

can, according to (3), be written in the form $\Sigma_{\omega}(\mathbf{r}) = \lambda_{\omega} C(\mathbf{r}, T)$, $\lambda_{\omega} = \lambda_{\omega_0} + h_{\omega}$, where $\lambda_{\omega_0} = \omega_0^2 / (\omega_0^2 + \omega^2)$ (such a representation of λ_{ω_0} follows from the form of equation (2)), ω_0 is the effective phonon frequency (for example, for Pb $\omega_0 \simeq 0.7\theta$, θ is the Debye temperature), $h_{\omega} \ll \lambda_{\omega}$ (for Pb $h(\pi T) \simeq 0.04$, $h(2\pi T) \ll h(\pi T)$).

For the function $C(\mathbf{r}, T)$ we obtain the equation

$$C(\mathbf{r}, T) = g^2 T \sum_{\omega'} \lambda_{\omega'} \int d\mathbf{l} \widetilde{G}_{-\omega'}^{(0)}(\mathbf{l}, \mathbf{r}) \widetilde{G}_{\omega'}^{(0)}(\mathbf{l}, \mathbf{r}) C(\mathbf{l}, T) - g^2 T \varphi(\mathbf{r}, T). \quad (4)$$

(The function $\varphi(\mathbf{r}, T)$ corresponds to the second term on the right-hand side of (3).)

We next substitute into the first term on the right-hand side of (4) the explicit expressions for the functions $\widetilde{G}_{\omega}^{(0)}(\mathbf{l}, \mathbf{r})$ (4) and, expanding the integrands in powers of $\mathbf{l} - \mathbf{r}$, perform the integration over $\mathbf{l}$. The subsequent summation over ω' is carried out without difficulty. It is essential that, since the consideration is carried out on the basis of the Fröhlich model (the Bardeen model is inapplicable in the presence of strong coupling), no logarithmic divergence arises, and we deal with convergent expressions. We then calculate the function $\varphi(\mathbf{r}, T)$, proportional to $\sim C^3(\mathbf{r}, T)$, just as in (4), and then eliminate the coupling constant g^2 from equation (4), introducing as a parameter the critical temperature T_{κ} .

After calculating, according to the formula

$$\mathbf{j}(\mathbf{r}) = \frac{ie}{m} (\nabla_{\mathbf{r}'} - \nabla_{\mathbf{r}})_{\mathbf{r}'=\mathbf{r}} T \sum_{\omega} \delta G_{\omega}(\mathbf{r}, \mathbf{r}'),$$

$$\delta G_{\omega} = \lambda_{\omega}^2 \int \tilde{G}_{\omega}^{(0)}(\mathbf{r}, \mathbf{l}) \tilde{G}_{-\omega}^{(0)}(\mathbf{m}, \mathbf{l}) \tilde{G}_{\omega}^{(0)}(\mathbf{m}, \mathbf{r}') C(\mathbf{l}) C(\mathbf{m}) d\mathbf{m} d\mathbf{l}$$

the density of the superconducting current, we finally arrive at the following system of Ginzburg-Landau equations, which thus prove to be valid also for describing the properties of superconductors with strong coupling:

$$\frac{1}{2m} \left(\frac{\partial}{\partial \mathbf{r}} + 2ie\mathbf{A} \right)^2 \psi + a \frac{T_{\kappa} - T}{T_{\kappa}} \psi - b|\psi|^2 \psi = 0, \quad (5)$$

$$\mathbf{j}(\mathbf{r}) = \frac{ie}{m} \left(\psi \frac{\partial \psi^*}{\partial \mathbf{r}} - \psi^* \frac{\partial \psi}{\partial \mathbf{r}} \right) - \frac{4e^2}{m} \mathbf{A} |\psi|^2. \quad (6)$$

Here

$$a = \frac{3R_1(\pi T)^2}{2B_1 \varepsilon_F}; \quad b = \frac{3D_1(\pi T)^2}{B_1 B_2 \varepsilon_F N};$$

$$B_1 = \sum_{n \geq 0} \frac{\omega_0^2 \lambda_{\omega}^0}{(\omega_0^2 + \omega'^2)(2n+1)^3}; \quad B_2 = \sum_{n \geq 0} \lambda_{\omega}^2 \frac{1}{(2n+1)^3};$$

$$D = \pi T \sum_{\omega'} \frac{\omega_0^2 \lambda_{\omega'}^3}{(\omega_0^2 + \omega'^2) \omega'^3};$$

$$R_1 = 1 + \frac{2}{3} \left(\frac{\pi T_{\kappa}}{\omega_0} \right)^2 + 2\pi \left[T \sum_{n'} \frac{\omega_0^2 h}{(\omega_0^2 + \omega'^2) \omega'} \Big|_{T=T_{\kappa}} - T \sum_{n'} \frac{\omega_0^2 h}{(\omega_0^2 + \omega'^2) \omega'} \Big|_{h \rightarrow T_{\kappa}} \right];$$

N is the density of the total number of electrons.

In deriving (5), (6), the substitution $\psi = C[NB_2/2(\pi T)^2]^{1/2}$ has been made. We note that for Pb $B_1 \simeq 0.81$, $B_2 \simeq 0.86$, $D_1 \simeq 0.64$.

Writing analogously to (4) the expression for the parameter κ of the Ginzburg-Landau theory, we find

$$\kappa = 0.96 \frac{\delta_L}{\xi_0} \left(\frac{D_1}{B_1 B_2} \frac{7\zeta(3)}{8} \right)^{1/2}$$

(δ_L is the London penetration depth at $T = 0^\circ$; ζ_0 is the pair size). For Pb we find $\chi \simeq 0.92 \delta_L / \zeta_0$. Using the parameter values δ_L and ζ_0 given in (1), we find $\chi_{\text{Pb}} \simeq 0.42$. We note that the known relation

$$\chi = 2.16 \cdot 10^7 \left| \frac{dH_k}{dT} \right|_{T_k} T_k \delta_L^2$$

leads, taking into account the thermodynamic relation

$$\frac{T_k}{4\pi} \left(\frac{dH_k}{dT} \right)_{T_k} = \beta C_n = C_s - C_n|_{T_k},$$

to the close value $\chi_{\text{Pb}} \simeq 0.48$. (For Pb $\delta_L = 3.9 \cdot 10^{-6}$ cm, $\gamma = 1.7 \cdot 10^3$ erg·deg⁻²·cm⁻³ (1, 5), $\beta_{\text{Pb}} = 2.4$ (3).)

We note that if, as $T \rightarrow T_k$, $\Delta(T) = a(1 - T/T_k)^{1/2}$ and $a_{\text{Pb}} \simeq 4.15$ –4.2, then $H_k/H_{k0}|_{\text{Pb}} - (1 - T^2/T_k^2)|_{T \rightarrow T_k} > 0$ in agreement with experimental data (for $T \rightarrow 0$, see (2)). For this value of a , the jump in heat capacity is $\beta_{\text{Pb}} \simeq 2.6$ in (3); $a_{\text{Pb}} = 4$, although, for example, allowance for the Coulomb interaction should increase a_{Pb} (see (3)).

The equations describing, in the case under consideration, the behavior of superconducting alloys can be obtained by a method analogous to that developed in (6). In this case we again arrive at equation (5), but with different values of the coefficients:

$$\frac{1}{2m} \left(\frac{\partial}{\partial r} + 2ie\mathbf{A} \right)^2 \psi + a_\tau \frac{T_k - T}{T_k} \psi - b_\tau |\psi|^2 \psi = 0. \quad (7)$$

Here

$$a_\tau = a/\chi_1(\rho); \quad b_\tau = b/[\chi_1(\rho)\chi_2(\rho)]^{1/2},$$

$$\chi_i(\rho) = \frac{B_{i\tau}}{B_i} \quad (i = 1, 2); \quad B_{1\tau} = \sum_{n \geq 0} \lambda_\omega \frac{1}{[1 + (2n + 1)^2(\pi T/\omega)^2](2n + 1 + \rho)(2n + 1)^2},$$

$$B_{2\tau} = \sum_{n \geq 0} \lambda_\omega^2 \frac{1}{(2n + 1 + \rho)(2n + 1)^2}; \quad \rho = (2\pi T\tau)^{-1},$$

τ is the “transport” time between collisions. In this case $\chi_{\text{alloy}} = \chi_{\text{pure}}/[\chi_1(\rho)\chi_2(\rho)]^{1/2}$. We note that the question of the fields H_{k2} and H_{k3} for Pb is discussed in (7).

The factors λ_ω determine a sharper convergence of the expressions B_i and $B_{i\tau}$ than in the usual case. With a sufficiently high degree of accuracy one may therefore take $\chi_{\text{alloy}} = \chi_{\text{pure}}(1 + \rho)$ or $\chi_{\text{alloy}} \simeq \chi_{\text{pure}}(1 + rn)$, where n is the impurity concentration and r is a proportionality coefficient. Thus, for Pb alloys, for example, a dependence $\Delta\chi = \chi_{\text{alloy}} - \chi_{\text{pure}} \sim n$ should be observed. In (8) (see also (9)) experimental values of χ_{alloy} are given for Pb+Tl and Pb+In alloys. Using the value obtained above, $\chi_{\text{Pb pure}} \simeq 0.42$, one can compare the theory with experiment. It turns out that for Pb alloys containing respectively 2.5, 5, and 15% thallium, and the alloys Pb+2% In and Pb+8% In, the dependence $\Delta\chi \sim n$ is indeed observed with a sufficient degree of accuracy.

2. Penetration depth in a weak electromagnetic field. We calculate, for $T \rightarrow T_k$, the penetration depth of a weak electromagnetic field into a superconductor with strong coupling. The Pippard kernel in the case under consideration, according to (10), has the form

$$K(T) = \pi T \sum_{\omega} \frac{\Sigma_{\omega}^2}{[\omega^2 + \Sigma_{\omega}^2]^{1/2}} \quad (8)$$

or, taking into account the relation given above for $\Sigma_{\omega}(T) = \lambda_{\omega}C(T)$,

$$K(T) = 2 \frac{C^2(T)}{(\pi T)^2} \sum_{n \geq 0} \left[\frac{\omega^2}{\omega^2 + (2n+1)^2(\pi T)^2} + h \right] \frac{1}{(2n+1)^3}.$$

For Pb, according to (3), $C(T) \simeq 4(1 - T/T_k)^{1/2}$, which gives $K(T)|_{T \rightarrow T_k} = b(1 - T/T_k)$, with $b_{\text{Pb}} = 2.7$ (we note that in the usual case $b = 2$).

The temperature dependence of the penetration depth $\delta(T)/\delta(0) = [K(0)/K(T)]^{1/2}$ for Pb turns out to have the form

$$\delta(T)/\delta(0)|_{T \rightarrow T_k} \simeq 0.58(1 - T/T_k)^{-1/2},$$

which differs noticeably from the dependence obtained in the usual case.

Formula (9), after substituting into it the value $\delta(0) = 3.9 \cdot 10^{-6}$, describes sufficiently well the experimental data presented in ⁽¹¹⁾, where it is noted that $\delta_{\text{Pb}} \simeq 2.3 \cdot 10^{-6}(1 - T/T_k)^{-1/2}$.

In the presence of impurities, use of the method developed in ⁽¹²⁾ leads to the expression

$$\delta = (4\pi Q)^{-1/2},$$

$$Q = \frac{Ne^2}{m} 2\pi T \sum_{n \geq 0} \frac{\Sigma_{\omega}^2}{(\omega^2 + \Sigma_{\omega}^2) [(\omega^2 + \Sigma_{\omega}^2)^{1/2} + 1/2\tau]}.$$

In the most interesting case of “dirty” alloys ($l \ll \xi_0$, l is the mean free path), we find, after substituting the expression for $\Sigma_\omega(T)$ at $T \rightarrow T_k$,

$$\delta|_{T \rightarrow T_k} = \frac{1}{4\pi\sqrt{\sigma} C(T)} \left[T \sum_{n \geq 0} \lambda_\omega^2 \omega^{-2} \right],$$

where σ is the conductivity of the normal metal. After summation we arrive at the following expression for the penetration depth in a Pb alloy containing a large concentration of impurity,

$$\delta_{\text{Pb}}|_{T \rightarrow T_k} = \frac{1}{3.25\pi} \left[2\sigma T_k \left(1 - \frac{T}{T_k} \right) \right]^{-1/2}.$$

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Note: Figure translations are in progress. See original paper for figures.

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