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Abstract

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PHYSICS

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CALCULATION OF THE DIFFUSION-SLIP VELOCITY OF A BINARY GAS MIXTURE

In work ⁽¹⁾, the diffusion-slip velocity of a binary gas mixture situated above a solid plane wall in the presence of a concentration gradient of one of the components tangential to the wall was calculated. The diffusion-slip velocity was obtained on the basis of the distribution function of gas molecules near the wall. In finding the distribution functions, the assumption, indicated already by Maxwell ⁽²⁾, was used that the distribution functions near the wall of molecules incident upon it do not differ from the distribution in the bulk of the gas. In reality, as a result of collisions of molecules incident on the wall with molecules reflected from the latter, the bulk distribution functions in the Knudsen layer are substantially distorted.

In our work the distribution functions are determined from model Boltzmann equations, which are linearized since the concentration gradient of one of the components tangential to the wall is small. The distribution functions of both incident and reflected molecules within the Knudsen layer depend on the distance to the wall.

Let us consider a binary gas mixture situated above a solid wall in a field of a concentration gradient of one of the components tangential to the wall. We choose the origin of coordinates on the surface of the wall. The x -axis is directed along the normal, and the y -axis along the surface. Let $\mathbf{v}_1$ and $\mathbf{v}_2$ be the velocities of molecules of the first and second components in a coordinate system fixed to the stationary wall. The model Boltzmann equations for the distribution functions of the first and second gases will have the form

$$v_{ix} \frac{\partial f_i}{\partial x} + v_{iy} \frac{\partial f_i}{\partial y} = \frac{f_i^{(0)} - f_i}{\tau_i}, \quad i = 1, 2, \quad (1)$$

where $f_i^{(0)}$ are local Maxwellian distribution functions for the mixture of gases moving with mass velocity $U(x)$. Using the smallness of $U(x)$ in comparison

with the velocities of thermal motion of the molecules, we may represent $f_i^{(0)}$ by the expressions

$$f_i^{(0)} = f_{i0} \left[1 + \frac{m_i}{kT} v_{iy} U(x) \right], \quad (2)$$

where $f_{i0} = n_i (m_i/2\pi kT)^{3/2} e^{-c_i^2}$.

To obtain a definite solution of system (1), it is necessary to prescribe boundary conditions at the wall. We shall assume that the molecules reflected from the wall have an isotropic Maxwellian velocity distribution

$$f_i^+(0, y, \mathbf{c}_i) = f_{i0}, \quad (3)$$

where $f_i^+ = f_i(x, y, \mathbf{c}_i)$ for $u_i > 0$, and $f_i^- = f_i(x, y, \mathbf{c}_i)$ for $u_i < 0$.

* $c_i = (m_i/2kT)^{1/2} v_i$ denotes the dimensionless molecular velocity, $c_i = (u_i; v_i; w_i)$.

We shall seek the solution in the form

$$f_i^\pm = f_{i0} [1 + \Psi_i(\infty, y, \mathbf{c}_i) + \Phi_i^\pm(x, c_i)], \quad (4)$$

where $f_{i0}(1 + \Psi_i)$ is the Chapman–Enskog distribution ⁽³⁾. The Ψ_i are given by the formulas:

$$\Psi_i = (-1)^i d_0 \frac{\rho_1 \rho_2}{\rho n_i \sqrt{m_i}} v_i \frac{\partial u_1}{\partial y}, \quad (5)$$

where $\rho_i = n_i m_i$, $\rho = \rho_1 + \rho_2$, $d_0 = \frac{2n}{n_1 n_2 \sqrt{2kT}} D_{12}$ (D_{12} is the diffusion coefficient).

We shall seek the correction to the distribution function in the form:

$$\Phi_i^\pm(x, \mathbf{c}_i) = v_i \varphi_i^\pm(x, u_i). \quad (6)$$

Substituting the distribution functions (4) into equations (1) and determining τ_i so that, at large distances from the wall, the distribution functions coincide with the Chapman–Enskog distribution, we obtain equations for $\varphi_i(x, u_i)$, defined by means of (6),

$$u_i \frac{\partial \varphi_i}{\partial x} + \theta_i \varphi_i(x, u_i) = 2\theta_i \left(\frac{m_i}{m_0} \right)^{1/2} G(x)^*, \quad (7)$$

where $G(x) = (m_0/2kT)^{1/2}U(x)$, $m_0 = m_1 + m_2$, $\theta_i = \rho\sqrt{m_i}/d_0\rho_1\rho_2$. The quantity $G(x)$ is expressed in terms of the functions φ_i as follows:

$$G(x) = \frac{1}{2\sqrt{\pi}} \sum_i \frac{\rho_i}{\rho} \left(\frac{m_0}{m_i}\right)^{1/2} \int_{-\infty}^{+\infty} \varphi_i e^{-u_i^2} du_i. \quad (8)$$

From (3) it is easy to obtain the boundary conditions for the functions φ_i :

$$\varphi_i^+(0, u_i) = (-1)^{i+1} d_0 \frac{\rho_1 \rho_2}{\rho n_i \sqrt{m_i}} \frac{\partial u_1}{\partial y} = C_i,$$

$$\varphi_i^\pm(\infty, u_i) = 2 \left(\frac{m_i}{m_0}\right)^{1/2} G(\infty). \quad (9)$$

Taking $G(x)$ as a known function, it is easy to write the solution of equations (7), with allowance for the boundary conditions (9),

$$\begin{aligned} \varphi_i^+ &= C_i \exp\left(-\frac{\theta_i x}{u_i}\right) + 2\theta_i \left(\frac{m_i}{m_0}\right)^{1/2} \exp\left(-\frac{\theta_i x}{u_i}\right) \int_0^x G(s) \exp\left(\frac{\theta_i s}{u_i}\right) \frac{ds}{u_i}, \\ \varphi_i^- &= -2\theta_i \left(\frac{m_i}{m_0}\right)^{1/2} \exp\left(\frac{\theta_i x}{u_i}\right) \int_x^\infty G(s) \exp\left(-\frac{\theta_i s}{u_i}\right) \frac{ds}{u_i}. \end{aligned} \quad (10)$$

Substituting (10) into expression (8), we obtain the integral equation for $G(x)$

$$G(x) = \frac{1}{2\sqrt{\pi}} \sum_i \left(\frac{m_0}{m_i}\right)^{1/2} \frac{\rho_i}{\rho} \left\{ C_i J_0(\theta_i x) + (11) + 2\theta_i \left(\frac{m_i}{m_0}\right)^{1/2} \int_0^x G(s) J_{-1}[\theta_i(x-s)] ds - 2\theta_i \left(\frac{m_i}{m_0}\right)^{1/2} \int_x^\infty G(s) J_1[\theta_i(x-s)] ds \right\}$$

where $J_n(x)$ are defined by the expressions ⁽⁴⁾

$$J_n(x) = \int_0^\infty t^n e^{-(t^2+x/t)} dt.$$

In what follows, for simplicity, we shall consider the particular case in which the density of one component is significantly greater than the density of the other, $n_2 \gg n_1$.

* The functions φ_i may be regarded as depending only on x and u_i , since $|\partial\varphi_i/\partial x| \gg |\partial\varphi_i/\partial y|$.

Let us integrate expression (11) by parts. Then, in order to give the equation a more convenient form, let us integrate both sides of the equation with respect to x from zero to ∞ ; taking into account that all terms containing x vanish as $x \rightarrow \infty$, we obtain the following integral equation for $dG(s)/ds$:

$$\int_0^\infty \frac{dG(s)}{ds} J_1(|x-s|) ds = -G(0)J_1(x) + \alpha_1 J_1\left(\sqrt{\frac{m_1}{m_2}}x\right) + \alpha_2 J_1(x), \quad (12)$$

where

$$\alpha_1 = \frac{d_0 n_1}{2n_2} \left(\frac{m_0 m_1}{m_2}\right)^{1/2} \frac{\partial n_1}{\partial y}, \quad \alpha_2 = -\frac{d_0 n_1}{2n_2} \left(\frac{m_0 m_1^2}{m_2^2}\right)^{1/2} \frac{\partial n_1}{\partial y}.$$

Introduce the functions $G_1(s)$ and $G_2(s)$ by means of the relation

$$G(s) = \alpha_1 G_1(s) + \alpha_2 G_2(s). \quad (13)$$

We choose $G_1(s)$ and $G_2(s)$ so that they satisfy the equations

$$\int_0^\infty \frac{dG_1(s)}{ds} J_1(|x-s|) ds = -G_1(0)J_1(x) + J_1\left(\sqrt{\frac{m_1}{m_2}}x\right) + J_2(x), \quad (14)$$

$$\int_0^\infty \frac{dG_2(s)}{ds} J_1(|x-s|) ds = -G_2(0)J_1(x) + J_1(x) - \frac{\alpha_1}{\alpha_2} J_2(x). \quad (15)$$

The solution of equations (14), (15) was indicated by Uhlenbeck ⁽⁵⁾, who used a method developed by Wiener ⁽⁶⁾. In ⁽⁵⁾ it is shown that the solution can be reduced to the form

$$\frac{dG_i(s)}{ds} = \begin{cases} \frac{1}{2\pi} \int_{-\infty}^{+\infty} z_i(\omega) e^{-i\omega s} d\omega, & s \geq 0, \\ 0, & s < 0, \end{cases} \quad (16)$$

$$z_i(\omega) = \int_{-\infty}^{+\infty} \frac{dG_i(s)}{ds} e^{i\omega s} ds. \quad (17)$$

The $z_i(\omega)$ are given by the expressions

$$z_1(\omega) = \int_0^\infty \frac{\exp(-y^2)}{\xi_1(\omega)} \left[\frac{y^3 - y^2 G_1(0)}{(1 - i\omega y)\xi_2(-i/y)} + \frac{y^2}{(a - i\omega y)\xi_2(-ia/y)} \right] dy, \quad (18)$$

$$z_2(\omega) = \int_0^\infty \frac{\exp(-y^2) \left[y^2(1 - G_2(0)) - \frac{\alpha_1}{\alpha_2} y^3 \right] dy}{\xi_1(\omega)(1 - i\omega y)\xi_2(-i/y)}, \quad (19)$$

where

$$\xi_n(\omega) = \exp \left\{ \frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{(-1)^n i \ln j_1(s) ds}{s - \omega} \right\}, \quad (20)$$

$$j_1(s) = \int_{-\infty}^{+\infty} J_1(|\eta|) e^{i\eta s} d\eta = 2 \int_0^\infty \frac{x^2 e^{-x^2} dx}{1 + s^2 x^2}, \quad a = \left(\frac{m_1}{m_2} \right)^{1/2}. \quad (21)$$

Unfortunately, these integrals cannot in the general case be evaluated analytically. However, it is not difficult to obtain two interesting results, namely the values $G_i(0)$ and $G_i(\infty)$.

It follows from expression (16) that, in order for the function dG_i/ds to be integrable, $z_i(\omega)$ must tend to zero as $\omega \rightarrow \infty$. Therefore, since $\xi_1(\omega)$ is of order $O(1/\omega)$ as $\omega \rightarrow \infty$, from (18) and (19) we obtain $G_i(0)$.

From relation (17), for $\omega = 0$ we obtain

$$z_i(\theta) = \int_0^\infty \frac{dG_i}{ds} ds = G_i(\infty) - G_i(0). \quad (22)$$

From the relations obtained, $G_i(\infty)$ are determined, after which it is easy to determine the slip velocity:

$$U_{sk} = \sqrt{2kT/m_0} G(\infty).$$

After simple calculations we obtain the formula for the diffusion slip velocity:

$$U_{sk} = \left[\frac{\mu(m_1/m_2)\sqrt{m_1/m_2} - m_1/m_2}{n_2} \right] D_{12} \frac{\partial n_1}{\partial y}.$$

μ is given by the expression

$$\mu = \frac{\beta_1}{\beta_2}(1 - \beta_3) + \beta_4,$$

where

$$\beta_1 = \int_0^\infty \frac{y \exp(-y^2) dy}{\xi_2(-ia/y)}, \quad \beta_2 = \int_0^\infty \frac{y \exp(-y^2) dy}{\xi_2(-il/y)}.$$

$$\beta_3 = \left(\frac{4}{\pi}\right)^{1/4} \int_0^\infty \frac{y^2 \exp(-y^2) dy}{\xi_2(-il/y)}, \quad \beta_4 = \left(\frac{m_2}{m_1}\right)^{1/2} \left(\frac{4}{\pi}\right)^{1/4} \int_0^\infty \frac{y^2 \exp(-y^2) dy}{\xi_2(-ia/y)}.$$

In the integrals β_i , $\xi_2(-ia/y)$ is expressed by means of the formula

$$\xi_2(-ia/y) = \exp\left(\frac{1}{2\pi} \int_{-\infty}^{+\infty} \frac{y \ln j_1(as) ds}{1 + s^2 y^2}\right).$$

The numerical calculation of the integrals carried out by us for the particular case $m_2/m_1 = 29/18$ gives the values $\beta_1 = 1.182$, $\beta_2 = 1.324$, $\beta_3 = 0.9921$, $\beta_4 = 1.132$.

For the case $m_2/m_1 = 29/18$ and $n_2 \gg n_1$, we obtain for the diffusion slip velocity the expression

$$U_{sk} = 0.277 D_{12} \frac{1}{n_2} \frac{\partial n}{\partial y}.$$

The result given by our formula is 1.66 times greater than that given by Brock's formula ⁽¹⁾.

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Note: Figure translations are in progress. See original paper for figures.

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