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1968

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Abstract

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UDC 531.382

Mechanics

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THE INFLUENCE OF MASS AND AERODYNAMIC ASYMMETRY OF A BODY ON THE CHARACTER OF ITS SPATIAL MOTION

(Presented by Academician A. A. Dorodnitsyn on 1 III 1968)

The motion of a body about its center of mass, moving uniformly and rectilinearly, is considered. It is assumed that the body has an axisymmetric shape, and that the distance from the center of mass to the axis of symmetry is $\overline{\Delta y} \neq 0$. The possibility and conditions are established for the occurrence, under the influence of mass asymmetry, of an increasing rotation of the body about the velocity vector (autorotation). The direction of the flight velocity $\mathbf{V}$ in the attached axes $Oxyz$ is specified by the direction cosines $^{(2)} \gamma_x, \gamma_y, \gamma_z$. The origin of coordinates coincides with the center of mass, the axis Ox is parallel to the symmetry axis of the shape, and the axis Oy lies in the plane center of mass–axis of symmetry of the shape.

The mass distribution is such that the centrifugal moments of inertia are small in comparison with the moments of inertia with respect to the axes Ox, Oy, Oz . The plane passing through the axis of symmetry and the velocity vector will be called the plane of the angle of attack $\tilde{\alpha}$. We note that γ_z, γ_y characterize the magnitudes of the sideslip and angle-of-attack angles.

The aerodynamic moments are written in the form

$$M_x = -\overline{\Delta y} \bar{c}_\alpha qSL \gamma_z; \quad M_y = -\bar{m}_\alpha qSL \gamma_z; \quad M_z = (\overline{\Delta y} c_\tau + \bar{m}_\alpha \gamma_y) qSL$$

Here $\bar{c}_\alpha = c_n(\tilde{\alpha})/\sin \tilde{\alpha} = f(\tilde{\alpha})$, $\bar{m}_\alpha = -m(\tilde{\alpha})/\sin \tilde{\alpha} = f_1(\tilde{\alpha}) \geq 0$, $c_\tau = c_\tau(\tilde{\alpha})$, $\tilde{\alpha} = \arccos \gamma_x$, where c_τ, c_n are the coefficients of the aerodynamic forces acting along the axis of symmetry and perpendicular to it; $m(\tilde{\alpha})$ is the coefficient of the moment of the rotating body in the plane $(Ox, \mathbf{V})$ for $\overline{\Delta y} = 0$.

The damping moments for small $\overline{\Delta y}$ may be regarded as the same as for $\overline{\Delta y} = 0$, and represented through the coefficients $m^{\omega_2}(\tilde{\alpha})$ (resistance to rotation of the body in the plane $\tilde{\alpha}$) and $m^{\omega_1}(\tilde{\alpha})$ (resistance to rotation about an axis lying in the plane $\tilde{\alpha}$ and perpendicular to Ox):

$$M_y(\vec{\omega}) = [(m^{\omega_2} + \overline{\Delta m_\omega} \gamma_y^2) \omega_y + \overline{\Delta m_\omega} \gamma_y \gamma_z \omega_z] qSL^2/V,$$

$$M_z(\vec{\omega}) = [(m^{\omega_2} + \overline{\Delta m_\omega} \gamma_z^2) \omega_z + \overline{\Delta m_\omega} \gamma_y \gamma_z \omega_y] qSL^2/V,$$

where $\overline{\Delta m_\omega} = (m^{\omega_1} - m^{\omega_2})/\sin^2 \tilde{\alpha}$ (as $\tilde{\alpha} \rightarrow 0$, $\overline{\Delta m_\omega}$ is a finite quantity).

The motion of the body is described by Euler's equations

$$\tilde{d}\mathbf{K}/dt + \vec{\omega} \times \mathbf{K} = \mathbf{M}; \quad \tilde{d}\vec{\gamma}/dt + \vec{\omega} \times \vec{\gamma} = 0, \quad (1)$$

where $\mathbf{M}$ is the principal moment of the external forces with respect to the center of mass; $\vec{\gamma} = \mathbf{V}/|\mathbf{V}|$ is the unit vector; $|\mathbf{V}|$ is the velocity of motion of the center of mass; $\mathbf{K} = I \cdot \vec{\omega}$ is the kinetic moment; I is the inertia tensor.

From equations (1) it follows that

$$\begin{aligned} \frac{d}{dt} \left(I_x \frac{\omega_x^2}{2} + I_y \frac{\omega_y^2}{2} + I_z \frac{\omega_z^2}{2} - I_{xy} \omega_x \omega_y - I_{yz} \omega_y \omega_z - I_{xz} \omega_x \omega_z \right) = \\ = \overline{\Delta y} c_\tau \omega_z qSL - \overline{\Delta y} \bar{c}_\alpha qSL \gamma_z \omega_x + \bar{m}_\alpha qSL \dot{\gamma}_x + \\ + \left[m^{\omega_1} \frac{(\omega_y \gamma_y + \omega_z \gamma_z)^2}{\gamma_y^2 + \gamma_z^2} + m^{\omega_2} \dot{\alpha}^2 \right] \frac{qSL^2}{V}, \end{aligned} \quad (2)$$

whence it is seen that for $\overline{\Delta y} \neq 0$, γ_z , $\omega_x = 0$ (plane motion) and sign-definite ω_z , the increment of kinetic energy

$$\Delta T = \overline{\Delta y} \oint_\alpha c_\tau(\alpha) qSL d\alpha$$

is positive for the corresponding sign of traversal in α and for sufficiently small $m^{\omega_1}, m^{\omega_2}$ (plane autorotation).

From equations (1) it also follows that

$$d(\mathbf{K} \cdot \vec{\gamma})/dt = \overline{\Delta y} (c_\tau - \bar{c}_\alpha \gamma_x) qSL \gamma_z + m^{\omega_1} (\omega_y \gamma_y + \omega_z \gamma_z) qSL^2/V. \quad (3)$$

Let us note that $(c_\tau - \bar{c}_\alpha \gamma_x) = c_y (\sin \tilde{\alpha})^{-1}$, where c_y is the lift coefficient. For γ_z not equal to zero on the average, the projection $(\mathbf{K} \cdot \vec{\gamma})$ may grow without bound when $\overline{\Delta y} c_y \neq 0$ (in particular, when $m^{\omega_1} = 0$).

Consider the special regime

$$\vec{\omega} = \Omega \vec{\gamma} + \varepsilon \vec{\omega}_1 = \Omega \bar{\vec{\gamma}} + \varepsilon \Omega \bar{\vec{\gamma}}_1 + \varepsilon \vec{\omega}_1, \quad (4)$$

where $\varepsilon \vec{\gamma}_1$, $\varepsilon \vec{\omega}_1$ are small vectors*, equal to zero on the time average; Ω is the mean value of the speed of rotation of the body about the velocity vector, and $\bar{\vec{\gamma}}$ is the position of the vector $\vec{\gamma}$ averaged over a unit interval of time. Using (4), we write

$$\frac{d(\mathbf{K} \cdot \vec{\gamma})}{dt} = \frac{d\mathbf{K}}{dt} \vec{\gamma} + \mathbf{K} \frac{d\vec{\gamma}}{dt} = \left[\dot{\Omega} (I \vec{\gamma}) + \Omega \left(I \frac{d\vec{\gamma}}{dt} \right) + \varepsilon (I \vec{\omega}_1) \right] \vec{\gamma} + [\Omega (I \vec{\gamma}) + \varepsilon (I \vec{\omega}_1)] \frac{d\vec{\gamma}}{dt}; \quad (5)$$

$$\frac{d\vec{\gamma}}{dt} = \vec{\gamma} \times \vec{\omega} = \varepsilon (\vec{\gamma} \times \vec{\omega}_1) = \varepsilon [(\bar{\vec{\gamma}} \times \vec{\omega}_1) + \varepsilon (\vec{\gamma}_1 \times \vec{\omega}_1)]. \quad (6)$$

Average (6) over time. Since

$$\frac{\varepsilon}{2T} \int_{-T}^{+T} \vec{\omega}_1 dt \sim \varepsilon^2,$$

then

$$\frac{d\vec{\gamma}}{dt} \sim \varepsilon^2, \quad d(\mathbf{K} \cdot \vec{\gamma})/dt = \dot{\Omega} [(I \vec{\gamma}) \cdot \vec{\gamma}] + O(\varepsilon^2).$$

Using the latter, we write the averaged equation of rotation of the body about the velocity vector

$$\left[I_x + I_y \frac{\bar{\gamma}_y^2}{\bar{\gamma}_x^2} + I_z \frac{\bar{\gamma}_z^2}{\bar{\gamma}_x^2} - 2I_{xy} \frac{\bar{\gamma}_y}{\bar{\gamma}_x} - 2I_{yz} \frac{\bar{\gamma}_y \bar{\gamma}_z}{\bar{\gamma}_x^2} - 2I_{xz} \frac{\bar{\gamma}_z}{\bar{\gamma}_x} \right] \dot{\omega}_x = \overline{\Delta y} \left(\frac{c_\tau}{\bar{\gamma}_x} - \bar{c}_a \right) qSL \bar{\gamma}_z + m \bar{\omega}_1 \frac{\bar{\gamma}_y^2 + \bar{\gamma}_z^2}{\bar{\gamma}_x^2} \bar{\omega}_x \frac{qSL^2}{V} \quad (7)$$

and the relations

$$\bar{\omega}_x = \Omega \bar{\gamma}_x, \quad \bar{\omega}_y = \Omega \bar{\gamma}_y, \quad \bar{\omega}_z = \Omega \bar{\gamma}_z.$$

Averaging the moment equations (1) over time, we obtain

$$\dot{\omega}_x \left(-I_{xy} + I_y \frac{\bar{\gamma}_y}{\bar{\gamma}_x} - I_{yz} \frac{\bar{\gamma}_z}{\bar{\gamma}_x} \right) = \bar{M}_{\Sigma y}; \quad (8,1)$$

$$\dot{\omega}_x \left(-I_{xz} - I_{yz} \frac{\bar{\gamma}_y}{\bar{\gamma}_x} + I_z \frac{\bar{\gamma}_z}{\bar{\gamma}_x} \right) = \bar{M}_{\Sigma z}, \quad (8,2)$$

* The vectors $\varepsilon \vec{\gamma}_1$, $\varepsilon \vec{\omega}_1$ are small if the oscillations of the angles of attack and slip are small, which can always

where

$$\bar{M}_{\Sigma y} = -\bar{m}_a q S L \bar{\gamma}_z - \left[\frac{\bar{\gamma}_z}{\bar{\gamma}_x} (I_x - I_z) - I_{xy} \frac{\bar{\gamma}_y \bar{\gamma}_z}{\bar{\gamma}_x^2} + I_{xz} \left(1 - \frac{\bar{\gamma}_z^2}{\bar{\gamma}_x^2} \right) + I_{yz} \frac{\bar{\gamma}_y}{\bar{\gamma}_x} \right] \bar{\omega}_x^2, \quad (9,1)$$

$$\bar{M}_{\Sigma z} = (\bar{\Delta y} c_\tau + \bar{m}_a \bar{\gamma}_y) q S L - \left[\frac{\bar{\gamma}_y}{\bar{\gamma}_x} (I_y - I_x) - I_{yz} \frac{\bar{\gamma}_z}{\bar{\gamma}_x} - I_{xy} \left(1 - \frac{\bar{\gamma}_y^2}{\bar{\gamma}_x^2} \right) + I_{xz} \frac{\bar{\gamma}_y \bar{\gamma}_z}{\bar{\gamma}_x^2} \right] \bar{\omega}_x^2, \quad (9,2)$$

where c_τ , $\bar{c}_a$, $\bar{m}_a$ are taken at the point $\bar{\gamma}_x = \sqrt{1 - \bar{\gamma}_y^2 - \bar{\gamma}_z^2}$. The left-hand side of equation (8,2) is a quantity of order ε^2 , since $\dot{\omega}_x \sim \varepsilon$, $I_{xz}/I_x \sim \varepsilon$, $\bar{\gamma}_z \sim \varepsilon$. Equating to zero the sum of the principal terms in (9,2), we obtain the first approximation for $\bar{\gamma}_y$ (independent of $\bar{\gamma}_z$):

$$(\bar{\Delta y} c_\tau + \bar{m}_a \bar{\gamma}_y) q S L = \left[\frac{\bar{\gamma}_y}{\bar{\gamma}_x} (I_y - I_x) - I_{xy} \left(1 - \frac{\bar{\gamma}_y^2}{\bar{\gamma}_x^2} \right) \right] \bar{\omega}_x^2. \quad (10)$$

Substituting $\dot{\omega}_x$ from (7) into (8,1), and taking (9,1) into account, for known $\bar{\gamma}_y$ we obtain an equation for $\bar{\gamma}_z$. Neglecting terms of order

$\varepsilon^2 \left(\bar{\gamma}_z^2 \ll \bar{\gamma}_x^2; \bar{\gamma}_z^2 \ll \bar{\gamma}_y^2; \frac{I_{ik}}{I_j} \frac{\bar{\gamma}_z}{\bar{\gamma}_x} = O(\varepsilon^2) \right)$, we write it in the form

$$\bar{\gamma}_z = - \frac{\left(I_{xz} + \frac{\bar{\gamma}_y}{\bar{\gamma}_x} I_{yz} \right) \bar{\omega}_x^2 + \Delta_0 \left[\frac{\bar{m}_a \bar{\gamma}_y^2}{\bar{\gamma}_x^2} \frac{q S L^2}{V} \bar{\omega}_x \right]}{\bar{m}_a q S L + \frac{\bar{\omega}_x^2}{\bar{\gamma}_x} \left[I_x - I_z - I_{xy} \frac{\bar{\gamma}_y}{\bar{\gamma}_x} \right] + \Delta_0 \bar{\Delta y} \left(\frac{c_\tau}{\bar{\gamma}_x} - \bar{c}_a \right) q S L}, \quad (11)$$

where

$$\Delta_0 = \left(\frac{\bar{\gamma}_y}{\bar{\gamma}_x} - \frac{I_{xy}}{I_y} \right) \left(\frac{I_x}{I_y} - 2 \frac{I_{xy} \bar{\gamma}_y}{I_y \bar{\gamma}_x} + \frac{\bar{\gamma}_y^2}{\bar{\gamma}_x^2} \right)^{-1}.$$

Fig. 1. Phase portrait of the motion for $\Delta y(c_\tau - \bar{c}_a \bar{\gamma}_x) \bar{\gamma}_z > 0$.
 $\delta = I_x - I_z - I_{xz} \bar{\gamma}_y / \gamma_x$. 1— $M_{x_0} > 0$, $\delta > 0$; 2— $M_{x_0} = 0$, $\delta < 0$; 3— $M_{x_0} = 0$, $\delta > 0$; 4— $M_{x_0} < 0$, $\delta > 0$

Figure 1: Fig. 1. Phase portrait of the motion for $\Delta y(c_\tau - \bar{c}_a \bar{\gamma}_x) \bar{\gamma}_z > 0$.
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If in (1) $\mathbf{M}$ contains the moment M_{x_0} , arising with a certain screw-like shape of the body and acting along the Ox axis, then the term M_{x_0} is added to the right-hand side of (7) and to the square bracket of the numerator of (11).

Equation (7), with (10), (11) taken into account, makes it possible to analyze a number of features of the body's rotation.

1. Let $M_{x_0} = 0$ and $\bar{m}^{\bar{\omega}_1} = 0$; then, for I_{xz} , $I_{yz} \neq 0$, $\bar{\Delta y} \neq 0$, and $\bar{\omega}_x(0) \neq 0$, the quantity $\bar{\gamma}_z \neq 0$. This leads, for example, when $\bar{\Delta y}(c_\tau / \bar{\gamma}_x - \bar{c}_a) \bar{\gamma}_z > 0$, to damping of $\bar{\omega}_x(t)$ for $\bar{\omega}_x(0) < 0$ and to an increase of $\bar{\omega}_x(t)$ for $\bar{\omega}_x(0) \geq 0$. Under a random disturbance $\bar{\omega}_x(0) \geq 0$, as $t \rightarrow \infty$ one will necessarily have $\bar{\omega}_x(t) \geq 0$.
2. As $\bar{\omega}_x^2$ increases, when $(I_x - I_z - I_{xy} \bar{\gamma}_y / \bar{\gamma}_x) < 0$, the denominator of expression (11) decreases and, for some $\bar{\omega}_x^*$, becomes zero. This is associated with loss of stability with respect to the sideslip angle ⁽¹⁾ ($\bar{\gamma}_z$, computed from (10), (11), increases without bound). For $\bar{\omega}_x$ close to $\bar{\omega}_x^*$, it is necessary to determine γ_y, γ_z by solving equations (8), (9) exactly.

A similar tendency also arises if $I_y - I_x > 0$ and $\bar{\omega}_x \rightarrow \bar{\omega}_x^{**}$ in determining γ_y from (10).

For $I_x > I_y, I_z$, no unbounded increase of $\bar{\gamma}_y, \bar{\gamma}_z$ occurs as $\bar{\omega}_x$ increases, since the rotation of a disk-like body about the axis of maxi-

of the small moment of inertia, is stable. In this case the quantity $\bar{\gamma}_z$ tends to a limit depending only on the mass distribution, and, correspondingly, for $m^\omega = 0$,

$$\dot{\bar{\omega}}_x(t) \rightarrow \lim_{\bar{\omega}_x \rightarrow \infty} \dot{\bar{\omega}}(\bar{\omega}_x).$$

Uniformly accelerated autorotation occurs, which in principle can be stopped by damping.

3. When damping is taken into account, in the region of small $\bar{\omega}_x$ there may exist a stability region (7). For small $\bar{\omega}_x$ the principal term will be the term proportional to the first power of $\bar{\omega}_x$. But when the quantity $|\bar{\omega}_x|$ exceeds a certain value $|\omega_{x1}|$, the influence of the term $\Delta y(c_\tau / \gamma_x - \bar{c}_a) \bar{\gamma}_z$ in (7) will be stronger than the influence of the damping term. As a result, the magnitude of the velocity $\bar{\omega}_x$ increases in the direction coinciding with the sign of ω_{x1} .

Fig. 1. Phase portrait of the motion for $\Delta y(c_\tau - \bar{c}_a \bar{\gamma}_x) \bar{\gamma}_z > 0$. $\delta = I_x - I_z - I_{xz} \bar{\gamma}_y / \gamma_x$.
 1— $M_{x_0} > 0$, $\delta > 0$; 2— $M_{x_0} = 0$, $\delta < 0$; 3— $M_{x_0} = 0$, $\delta > 0$; 4— $M_{x_0} < 0$, $\delta > 0$.

4. For a sufficiently large moment M_{x_0} and

$$M_{x_0} \Delta y(c_\tau / \gamma_x - \bar{c}_a) \bar{\gamma}_z > 0,$$

the product $\dot{\bar{\omega}}_x \cdot M_x$ is positive for all $\bar{\omega}_x$ (9). For

$$M_{x_0} \Delta y(c_\tau / \gamma_x - \bar{c}_a) \bar{\gamma}_z < 0,$$

the phase trajectory $\dot{\bar{\omega}}(\bar{\omega}_x)$ (Fig. 1) intersects the axis $\dot{\bar{\omega}}_x = 0$ twice. The intersections determine the speed of established rotation $\bar{\omega}_x^{(s)}$ and the critical value $\bar{\omega}_x^{(c)}$, which determines the boundary of the autorotation region on the $\bar{\omega}_x$ axis.

5. The consideration given for the case $q = \text{const}$ is also valid for $q = q(t)$, if (4) is valid. Then, for $M_{x_0} = M_{x_0}(q)$, the values $\bar{\omega}_x^{(s)}, \bar{\omega}_x^{(c)}$ (item 4), as well as $\bar{\omega}_x^*$ and $\bar{\omega}_x^{**}$ (item 2), will depend on time, and a loss of stability may occur, associated with the relative change of the quantities $\bar{\omega}_x, \bar{\omega}_x^*, \bar{\omega}_x^{**}, \bar{\omega}_x^{(s)}, \bar{\omega}_x^{(c)}$.

Thus, it has been shown that, in the motion of an axisymmetric body having $\Delta y \neq 0$, under the influence of the peculiarities of the mass distribution, a progressive increase of rotation about the velocity vector (autorotation) may arise.

Received

14 II 1968

CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.

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