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Abstract

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THEORY OF ELASTICITY

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SUPERSONIC FLUTTER OF THREE-LAYER PLATES

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1°. The investigations presented below are based on a system of differential equations of elastic equilibrium for three-layer plates, obtained in ⁽¹⁾. In ^(2, 3) a number of exact solutions of boundary-value problems for these equations were given for the case of free vibrations of a three-layer beam (an infinitely wide plate) in a vacuum and in a gas flow. In the present paper the investigations concern the behavior of three-layer plates of finite elongation in a supersonic gas flow.

The differential equation of elastic equilibrium of a three-layer plate in a gas flow, in the absence of shear forces, takes the form ⁽¹⁾

$$D(1-\vartheta h^2\beta^{-1}\nabla^2)\nabla^2\nabla^2\chi - N_x\frac{\partial^2}{\partial x^2}(1-h^2\beta^{-1}\nabla^2)\chi - \Omega\frac{\partial^2}{\partial t^2}(1-h^2\beta^{-1}\nabla^2)\chi - N_y\frac{\partial^2}{\partial y^2}(1-h^2\beta^{-1}\nabla^2)\chi - q = 0. \quad (1)$$

Here $\chi(x, y, t)$ is a displacement function related to the deflection $w(x, y, t)$ by the relation

$$w = (1 - h^2\beta^{-1}\nabla^2)\chi, \quad (1, 2)$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$.

The quantities D , ϑ , β^{-1} , Ω characterize, respectively, the cylindrical stiffness of the three-layer package, the bending stiffness of the load-bearing layers, the shear stiffness of the filler, and the mass per unit area of the three-layer package ⁽¹⁾; h is the package thickness; N_x , N_y are the compressive (tensile) forces in the longitudinal and transverse directions; $q(x, y, t)$ is the transverse aerodynamic load.

2°. Suppose that the plate is flowed over on one side by a supersonic gas flow directed along the x -axis. We assume that two edges of the plate parallel to the velocity vector $\vec{V}$ of the incident flow are hinged, while the other two edges, normal to the vector $\vec{V}$, have arbitrary fixing.

We represent a particular solution of (1,1) in the form

$$\chi(x, y, t) = \sin \frac{n\pi y}{b} \chi(x) e^{i\omega t}, \quad (2, 1)$$

where n is the number of half-waves in the transverse direction; b is the width of the plate; $\chi(x)$ is a complex function of the real variable x ; ω is the complex frequency of vibrations; t is time.

We assume that the aerodynamic load can be calculated in the linear approximation of piston theory

$$q = -\frac{\chi p_0}{a_0} \left(\frac{\partial w}{\partial t} + V \frac{\partial w}{\partial x} \right), \quad (2, 2)$$

where p_0 , a_0 are the static pressure and speed of sound in the undisturbed flow, and $\chi = c_p/c_v$ is the ratio of the corresponding specific heats.

Substituting (2.1), (2.2) into (1.1) and passing to the dimensionless coordinates x/a , y/b (where a is the length of the plate) and to the dimensionless time $\tau = t/\sqrt{\Omega D}$, we obtain

$$a_6 \frac{d^6 \chi}{dx^6} + a_4 \frac{d^4 \chi}{dx^4} + a_3 \frac{d^3 \chi}{dx^3} + a_2 \frac{d^2 \chi}{dx^2} + a_1 \frac{d\chi}{dx} + a_0 \chi = 0, \quad (2.3)$$

where

$$\begin{aligned} a_6 &= -\vartheta k; \quad a_4 = 1 + 3\vartheta k n^2 \pi^2 \bar{a}^2 + k n_x; \\ a_3 &= -k p_*; \quad -a_2 = 2n^2 \pi^2 \bar{a}^2 + 3\vartheta k n^4 \pi^4 \bar{a}^4 + (1 + k n^2 \pi^2 \bar{a}^2) n_x + \\ &\quad + k n^2 \pi^2 \bar{a}^2 n_y - k \omega_*^2 + i k \varepsilon \omega_*; \\ a_1 &= p_* (1 + k n^2 \pi^2 \bar{a}^2); \quad a_0 = (1 + \vartheta k n^2 \pi^2 \bar{a}^2) n^4 \pi^4 \bar{a}^4 + \\ &\quad + (1 + k n^2 \pi^2 \bar{a}^2) n^2 \pi^2 \bar{a}^2 n_y - (1 + k n^2 \pi^2 \bar{a}^2) \omega_*^2 + i \varepsilon \omega_* (1 + k n^2 \pi^2 \bar{a}^2). \end{aligned} \quad (2.4)$$

Here

$$k = h^2 \beta^{-1} / a^2; \quad n_x = N_x a^2 / D; \quad n_y = N_y a^2 / D; \quad \bar{a} = a/b;$$

$$p_* = \chi p_0 a^3 M / D; \quad M = V / a_0; \quad \varepsilon = \chi p_0 a^2 / \sqrt{\Omega D}; \quad \omega_* = \omega a^2 \sqrt{\Omega / D} \quad (2.5)$$

are dimensionless parameters.

Let a particular solution of (2.3) have the form

Fig. 1. Variation of the vibration frequency ω_* and the critical flutter speed p_{*f} of a three-layer plate as functions of the tension n_x ; $\vartheta = 0.05$, $k = 1$, $\bar{a} = 1$, $n = 1$, $\varepsilon = 0.1$ (simply supported edges, first two frequencies)

Figure 1: Fig. 1. Variation of the vibration frequency ω_* and the critical flutter speed p_{*f} of a three-layer plate as functions of the tension n_x ; $\vartheta = 0.05$, $k = 1$, $\bar{a} = 1$, $n = 1$, $\varepsilon = 0.1$ (simply supported edges, first two frequencies)

$$\chi(x) = e^{i\alpha x}, \quad (2.6)$$

where α is a root of the algebraic equation

$$-a_6\alpha^6 + a_4\alpha^4 - ia_3\alpha^3 - a_2\alpha^2 + ia_1\alpha + a_0 = 0. \quad (2.7)$$

The general solution of (2.3) will be

$$\chi(x) = \sum_{j=1}^6 C_j e^{i\alpha_j x}, \quad (2.8)$$

where C_j are constants to be determined from the boundary conditions of the problem.

Fig. 1. Variation of the vibration frequency ω_* and the critical flutter speed p_{*f} of a three-layer plate as functions of the tension n_x ; $\vartheta = 0.05$, $k = 1$, $\bar{a} = 1$, $n = 1$, $\varepsilon = 0.1$ (simply supported edges, first two frequencies)

The solution of the original differential equation (1.1) for a certain fixed number n can be represented in the form

$$\chi(x, y, t) = \sin \frac{n\pi y}{b} \cdot e^{i\omega t} \sum_{j=1}^6 C_j e^{i\alpha_j x}. \quad (2.9)$$

3°. Consider a number of boundary-value problems.

- a) Simply supported along all four edges. The boundary conditions for the displacement function $\chi(x)$ at $x = 0$, $x = 1$ and $y = 0$, $y = 1$ have the form $\begin{pmatrix} 1, & 2 \end{pmatrix}$

$$\chi - \nabla^2 \chi = \nabla^2 \nabla^2 \chi = 0. \quad (3.1)$$

Substituting (2.8) into (3.1), we obtain a system of homogeneous linear algebraic equations for the coefficients C_j . The condition for a nontrivial solution of the system will be

Fig. 2

Figure 2: Fig. 2

Fig. 3

Figure 3: Fig. 3

$$\Delta/\delta = 0, \quad (3.2)$$

where Δ is the determinant of the system; δ is the Vandermonde determinant formed from the roots α_j . The numerator of (3.2) is a sum of 10 terms Δ_j of the form

$$\Delta_j = A_j B_j \quad (j = 1, 2, \dots, 10), \quad (3.3)$$

where

$$\begin{aligned} A_1 / (e^{\alpha_1 + \alpha_2 + \alpha_3} - e^{-(\alpha_1 + \alpha_2 + \alpha_3)}) &= (123) + (231) + (312), \\ B_1 &= (546) + (465) + (654); \end{aligned} \quad (3.4)$$

$$\begin{aligned} (123) &= (\alpha_2^2 - \alpha_1^2)(\alpha_1^2 - n^2 \pi^2 \bar{a}^2) \times \\ &\quad \times (\alpha_2^2 - n^2 \pi^2 \bar{a}^2), \quad (3.5) \\ (546) &= (\alpha_4^2 - \alpha_5^2)(\alpha_5^2 - n^2 \pi^2 \bar{a}^2) \times \\ &\quad \times (\alpha_4^2 - n^2 \pi^2 \bar{a}^2). \end{aligned}$$

Fig. 2. Effect of tension n_y on the natural-frequency ω_* of a three-layer plate with hinged supported edges, $n_x = -3.25$, $\vartheta = 0.05$, $k = 1$, $\bar{a} = 1$, $n = 1$, $\varepsilon = 0.1$ (the first two frequencies)

The remaining terms of the sum, $j = 2, 3, \dots$, are obtained from (3.4) by the corresponding permutation of the indices of the roots α_j .

b) Hinged support along all four edges, but at the edges $x = 0$, $x = 1$ there are diaphragms rigid in the plane of the edge, preventing

Fig. 3. Variation of the critical divergence and flutter speed of a three-layer plate as a function of the tensions n_x, n_y ; $\vartheta = 0.05$, $k = 1.00$, $\bar{a} = 1$, $n = 1$, $\varepsilon = 0.1$ (hinged supported edges, first two harmonics)

shear of the load-bearing layers. The boundary conditions for $x = 0$, $x = 1$ will be

$$\left(\frac{\partial^2}{\partial x^2} + \nu \frac{\partial^2}{\partial y^2} \right) (1 - \vartheta k \nabla^2) \chi = (1 - k \nabla^2) \chi = \frac{\partial}{\partial x} \nabla^2 \chi = 0, \quad (3.6)$$

Fig. 4. Effect of elongation on the critical flutter speed of a three-layer plate with hinged edges: $\vartheta = 0.05$, $k = 1.00$, $n = 1$

Figure 4: Fig. 4. Effect of elongation on the critical flutter speed of a three-layer plate with hinged edges: $\vartheta = 0.05$, $k = 1.00$, $n = 1$

where ν is the generalized Poisson's ratio ⁽¹⁾. The components of the first term Δ_j ($j = 1, 2, \dots, 20$) will take the form

$$(123) = (\alpha_2 - \alpha_1)(\alpha_3^2 - \nu n^2 \pi^2 \bar{a}^2)[1 - \vartheta k(\alpha_3^2 - n^2 \pi^2 \bar{a}^2)] \times [\alpha_2^2 + \alpha_2 \alpha_1 + \alpha_1^2 - n^2 \pi^2 \bar{a}^2 - k(\alpha_1^2 - n^2 \pi^2 \bar{a}^2)(\alpha_2^2 - n^2 \pi^2 \bar{a}^2)].$$

Similarly to (546), but with the factor $\exp(\alpha_4 + \alpha_5 + \alpha_6)$.

- c) Hinged support along the edges $x = 0$, $x = 1$ and clamping along the edges $y = 0$, $y = 1$. The boundary conditions for the clamped edge are

$$(1 - k \nabla^2) \chi = \partial \chi / \partial x = \nabla^2 \partial \chi / \partial x = 0. \quad (3, 8)$$

For this case ($j = 1, 2, \dots, 10$)

$$(123) = \alpha_1 \alpha_2 [1 - k(\alpha_3 - n^2 \pi^2 \bar{a}^2)] (\alpha_2 - \alpha_1). \quad (3, 9)$$

- d) Two edges $y = 0$, $y = 1$ are hinged, and the other two, $x = 0$, $x = 1$, are free. The boundary conditions for the free edges are

$$\left(\frac{\partial^2}{\partial x^2} + \nu \frac{\partial^2}{\partial y^2} \right) \chi = \left(\frac{\partial^2}{\partial x^2} + \nu \frac{\partial^2}{\partial y^2} \right) \nabla^2 \chi = \left[\frac{\partial^3}{\partial x^3} + (2 - \nu) \frac{\partial^3}{\partial x \partial y^2} \right] (1 - \vartheta k \nabla^2) \chi = 0. \quad (3, 10)$$

In this case ($j = 1, 2, \dots, 20$)

$$(123) = \alpha_3 [\alpha_3^2 - (2 - \nu) n^2 \pi^2 \bar{a}^2] [1 - \vartheta k(\alpha_3^2 - n^2 \pi^2 \bar{a}^2)] (\alpha_1^2 - \nu n^2 \pi^2 \bar{a}^2) (\alpha_2^2 - \nu n^2 \pi^2 \bar{a}^2) (\alpha_2^2 - \alpha_1^2). \quad (3, 11)$$

Fig. 4. Effect of elongation on the critical flutter speed of a three-layer plate with hinged edges: $\vartheta = 0.05$, $k = 1.00$, $n = 1$.

From an analysis of the results of the calculation (Figs. 1-4) it follows that the dimensionless flutter speed of a three-layer plate p_* depends strongly on the magnitude of the longitudinal load n_x (Fig. 1) and on the elongation $\bar{a}$ (Fig. 4), and depends practically not at all on the magnitude of the transverse load n_y (Fig. 2). However, the latter has a substantial effect on the divergence speed and, as a consequence, on the size of the stability region (Fig. 3). The effect

of aerodynamic damping becomes noticeable only for sufficiently large values of the coefficient ε (Fig. 3).

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REFERENCES

1. E. I. Grigolyuk, P. P. Chulkov, *Izv. AN SSSR, Mekhanika i mashinostr.*, No. 1, 67 (1964).
2. A. I. Smirnov, *DAN*, **172**, No. 3, 561 (1967).
3. A. I. Smirnov, *DAN*, **180**, No. 5 (1968).

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