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Abstract

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MATHEMATICS

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ON THE LYAPUNOV-GÜNTER THEOREMS FOR SPECIAL HEAT POTENTIALS

(Presented by Academician S. L. Sobolev on 24 V 1967)

In paper ⁽¹⁾ (see also ^(2, 3)) a theory was constructed for the smoothness of heat potentials of a simple and a double layer with densities distributed on noncylindrical surfaces, entirely analogous to the classical Lyapunov-Günter theory (see ⁽⁴⁾) for harmonic potentials. In ⁽⁵⁾ (see also ⁽⁶⁾) the construction of a similar theory was begun for the special heat potentials P and Q (introduced by M. Pani in ⁽⁷⁾), which play an important role in the theory of boundary-value problems with an oblique derivative for a parabolic equation of the second order. The present note, consisting of two sections, contains a complete and systematic investigation in Hölder smoothness spaces of the special heat potential of a simple layer $P[\varphi]$ and the heat potential $Q[\varphi]$, which is the derivative of the potential $P[\varphi]$ in an oblique direction. In § 1 the improving properties of the direct values $Q[\varphi]$ on noncylindrical surfaces of various smoothness are studied, while in § 2 the smoothness of the heat potential $P[\varphi]$ in the closed domain $\bar{D}_T$, having Γ as its lateral boundary, is studied. The notation and definitions of the author's papers ^(1, 5) are used in the note.

Let D_T be a bounded domain of the $(n + 1)$ -dimensional Euclidean space $(x, t) \equiv (x_1, x_2, \dots, x_n; t)$, situated between two hyperplanes $t = 0$ and $t = T > 0$ and having an n -dimensional noncylindrical surface Γ as its lateral boundary. Consider the heat potentials

$$P(\bar{x}, t) \equiv P[\varphi] \equiv \int_0^t d\tau \iint_{\Gamma_\tau} P(\bar{x}, t; y, \tau) \varphi(y, \tau) d\sigma_y(\tau), \quad (\bar{x}, t) \in D_T,$$

$$Q(\bar{x}, t) \equiv Q[\varphi] \equiv \partial P(\bar{x}, t) / \partial \nu(x, t), \quad (x, t) \in \Gamma_t;$$

Γ_τ is the section of the surface Γ by the hyperplane $t = \tau$, on which a field of directions is given with unit vector

$\nu(x, t) = \{\nu_1(x, t), \dots, \nu_n(x, t; 0)\}$, lying in the section $\Omega_t \equiv D_T \cap \{t = t\}$ and making an acute angle not exceeding $\pi/2 - d_0$ ($d_0 > 0$) with the normal $N(x, t)$

interior with respect to Ω_t at the point $(x, t) \in \Gamma_t$. $P(\bar{x}, t; y, \tau)$ is the special fundamental solution, introduced by M. Pani in ⁽⁷⁾, of the n -dimensional heat-conduction equation corresponding to the field of oblique directions $\nu(x, t)$.

§ 1. Improving properties of the direct values of the heat potential $\bar{Q}[\varphi]$. The notation $\alpha', \alpha^0, \alpha^*, \beta'$ from ⁽⁵⁾ will be used.

Theorem 1. Suppose that for Γ , ν , and φ the following conditions are satisfied: Γ is of type

$$\Pi_{2m+1, 1, (1+\alpha)/2}^{m+1, \alpha, \alpha/2}, \quad 0 < \alpha \leq 1,$$

$$\nu_j \in H_{2m+1, \beta, \beta/2}^{m, 1, (1+\beta)/2}(\Gamma), \quad 0 < \alpha \leq \beta \leq 1,$$

$$\cos(\nu(x, t), N(x, t)) \geq d > 0, \quad (x, t) \in \Gamma_t, \quad d = \text{const}, \quad 0 \leq t \leq T, \quad (1)$$

$$\varphi \in H_{2m-1, 1, (1+\alpha)/2}^{m, \alpha, \alpha/2}(\Gamma),$$

where

$$\begin{aligned} |\partial^k \varphi(y, \tau) / \partial \tau^k| &\leq |\partial^m \varphi / \partial t^m|_{\alpha} \tau^{m-k+\alpha/2}, \\ (y, \tau) &\in \Gamma, \quad k = 0, 1, 2, \dots, m. \end{aligned}$$

Then for $m = 1, 2, \dots$ (for $m = 0$ see Lemma 4 (5)) for $\bar{Q}(x, t)$ one has

$$\bar{Q} \in H_{2m+1, \alpha, \alpha^*/2}^{m, 1, (1+\alpha^*)/2}(\Gamma),$$

where the Hölder constants have the form $(C)|\varphi|_{2m+\alpha}$,

$$|\partial^k \bar{Q}(x, t) / \partial t^k| \leq (C)|\varphi|_{2m+\alpha} t^{m-k+(1+\alpha)/2}, \quad (x, t) \in \Gamma, \quad k = 0, 1, \dots, m,$$

and when

$$\begin{aligned} p = 0, 1, 2, \dots, m, \quad l_j = 0, 1, \dots, 2(m-p), \quad j = 1, 2, \dots, k, \\ \sum_{j=1}^k l_j = 2(m-p), \end{aligned} \quad (2)$$

where $k = n - 1$,

$$\left| \partial^{2m-p+l} \overline{Q}(x, t) / \partial t^p \partial x_1^{l_1} \dots \partial x_{n-1}^{l_{n-1}} \partial x_i^l \right| \leq (C) |\varphi|_{2m+\alpha} t^{(1+\alpha-l)/2}, \quad l = 0, 1.$$

Theorem 2. Suppose that for Γ , ν , and φ the following conditions are satisfied:
 Γ is of type

$$\mathcal{L}_{2m+3, \alpha, \alpha/2}^{m+1, 1, (1+\alpha)/2}, \quad 0 < \alpha \leq 1 \quad (m = 0, 1, 2, \dots),$$

$$\nu_j \in H_{2m+1, 1, (1+\beta)/2}^{m+1, \beta, \beta/2}(\Gamma), \quad 0 < \alpha \leq \beta \leq 1, \quad j = 1, 2, \dots, m$$

(with (1) satisfied),

$$\varphi \in H_{2m+1, \alpha, \alpha/2}^{m, 1, (1+\alpha)/2}(\Gamma),$$

$$\left| \partial^{k+l} \varphi(y, \tau) / \partial \tau^k \partial y_i^l \right| \leq |\partial^m \varphi / \partial t^m|_{1+\alpha} \tau^{m-k+(1+\alpha-l)/2}$$

$$(y, \tau) \in \Gamma, \quad k = 0, 1, 2, \dots, m; \quad i = 1, 2, \dots, n-1; \quad l = 0 \text{ for } k < m, \\ l = 0, 1 \text{ for } k = m.$$

Then for $m = 0, 1, 2, \dots$ for $\overline{Q}(x, t)$ one has

$$\overline{Q} \in H_{2m+1, 1, (1+\alpha^*)/2}^{m+1, \alpha^*, \alpha^*/2}(\Gamma),$$

where the Hölder constants have the form $(C) |\varphi|_{2m+1+\alpha}$,

$$\left| \partial^k \overline{Q}(x, t) / \partial t^k \right| \leq (C) |\varphi|_{2m+1+\alpha} t^{m-k+\alpha/2}, \quad k = 0, 1, \dots, m,$$

and, when (2) is satisfied, where $k = n-1$,

$$\left| \partial^{2m+1-p} \overline{Q}(x, t) / \partial t^p \partial x_1^{l_1} \dots \partial x_{n-1}^{l_{n-1}} \partial x_i \right| \leq (C) |\varphi|_{2m+1+\alpha} t^{(1+\alpha)/2},$$

$$\left(\left| \partial^{2m+1-p} \overline{Q}(x, t) / \partial t^{p+1} \partial x_1^{l_1} \dots \partial x_{n-1}^{l_{n-1}} \right|, \right.$$

$$\left. \left| \partial^{2m+2-p} \overline{Q}(x, t) / \partial t^p \partial x_1^{l_1} \dots \partial x_{n-1}^{l_{n-1}} \partial x_i \partial x_j \right| \right) \leq (C) |\varphi|_{2m+1+\alpha} t^{\alpha/2}.$$

§ 2. Smoothness of the special heat potential of a simple layer $P[\varphi]$ in the closed domain $\overline{D}_T$

Theorem 3. Suppose that for Γ , ν the conditions of Theorem 1 are satisfied,

$$\varphi \in H_{2m+1, \alpha, \alpha/2}^{m, 1, (1+\alpha)/2}(\Gamma) \quad \text{for } m = 0, 1, 2, \dots,$$

$$\partial^k \varphi(y, 0) / \partial \tau^k \equiv 0, \quad k = 0, 1, \dots, m, \quad (y, 0) \in \Gamma_0,$$

and, when (2) is satisfied, where $k = n - 1$,

$$\left| \partial^{2m-p+l} \varphi(y, \tau) / \partial \tau^p \partial y_1^{l_1} \dots \partial y_{n-1}^{l_{n-1}} \partial y_i^l \right| \leq (C) |\varphi|_{2m+1+\alpha} \tau^{(1+\alpha-l)/2}, \quad l = 0, 1.$$

Then

$$P \in H_{2m+1, 1, (1+\alpha)/2}^{m+1, \alpha', \alpha'/2}(\overline{D}_T),$$

where the Hölder constants have the form $(C) |\varphi|_{2m+1+\alpha}$, and (see (2), where $k = n$)

$$\left| \partial^{2m+1+l-p} P(\bar{x}, t) / \partial t^p \partial \bar{x}_1^{l_1} \dots \partial \bar{x}_n^{l_n} \partial \bar{x}_i \right| \leq (C) |\varphi|_{2m+1+\alpha} t^{(1+\alpha-l)/2}, \quad l = 0, 1,$$

$$\left| \partial^{2m+1-p} P(\bar{x}, t) / \partial t^{p+1} \partial \bar{x}_1^{l_1} \dots \partial \bar{x}_n^{l_n} \right| \leq (C) |\varphi|_{2m+1+\alpha} t^{\alpha/2}, \quad (\bar{x}, t) \in \overline{D}_T$$

(for $m = 0$, Theorem 3 coincides with Lemma 6 of (5)).

Theorem 4. Suppose that for Γ and ν the conditions of Theorem 2 are fulfilled, and

$$\varphi \in H_{2m+1, 1, (1+\alpha)/2}^{m+1, \alpha, \alpha/2}(\Gamma) \quad \text{for } m = 0, 1, 2, \dots,$$

where

$$\partial^k \varphi(y, 0) / \partial \tau^k = 0, \quad k = 0, 1, 2, \dots, m+1,$$

and (see (2), where $k = n - 1$)

$$\left| \partial^{2m+1+l-p} \varphi(y, \tau) / \partial \tau^p \partial y_1^{l_1} \dots \partial y_{n-1}^{l_{n-1}} \partial y_i \partial y_j \right| \leq (C) |\varphi|_{2m+2+\alpha} \tau^{(1+\alpha-l)/2}, \quad l = 0, 1;$$

$$\left| \partial^{2m+1-p} \varphi(y, \tau) / \partial \tau^{p+1} \partial y_1^{l_1} \dots \partial y_{n-1}^{l_{n-1}} \right| \leq (C) |\varphi|_{2m+2+\alpha} \tau^{\alpha/2}.$$

Then

$$P \in H_{2m+3, \alpha', \alpha'/2}^{m+1, 1, (1+\alpha)/2}(\overline{D}_T),$$

where the Hölder constants have the form $(C)|\varphi|_{2m+2+\alpha}$, and, when (2) is fulfilled, where $k = n$ and m is replaced by $m + 1$,

$$\left| \partial^{2(m+1)+l-p} P(\bar{x}, t) / \partial t^p \partial \bar{x}_1^{l_1} \dots \partial \bar{x}_n^{l_n} \partial \bar{x}_i \right| \leq (C) |\varphi|_{2m+2+\alpha} t^{(1+\alpha-l)/2}, \quad l = 0, 1.$$

The proofs of Theorems 1-4 are carried out by the methods of the papers (², ³, ⁶).

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