

GREEN' S FUNCTIONS FOR QUANTUM FERMI SYSTEMS OF CHARGED PARTICLES WITH COLLISIONS

V. N. Mel'nikov

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Abstract

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PHYSICS

V. N. MEL' NIKOV

GREEN'S FUNCTIONS FOR QUANTUM FERMION SYSTEMS OF CHARGED PARTICLES WITH COLLISIONS

(Presented by Academician N. N. Bogolyubov, 4 V 1967)

Charged Fermi particles are considered, interacting according to Coulomb's law and situated in a compensating field of charge of the opposite sign. We shall start from equations for quantum distribution functions of the form

$$D_1(t, x_1, x'_1) = \langle \Psi^+(t, x'_1); \Psi(t, x_1) \rangle,$$

where $\langle \dots \rangle = Q^{-1} \text{Sp}(\dots e^{-(H-\lambda N)/\theta})$; $Q = \text{Sp} e^{-(H-\lambda N)/\theta}$, λ is the chemical potential; $x = \{\bar{r}, \sigma\}$,

$$\Psi(t, x) = \frac{1}{(2\pi)^{3/2}} \int e^{ipx} a_{p\sigma} dp, \quad \Psi^+(t, x) = \frac{1}{(2\pi)^{3/2}} \int e^{-ipx} a_{p\sigma}^+ dp.$$

Then

$$\begin{aligned} D_1(t, x_1, x'_1) &= \frac{1}{(2\pi)^3} \iint e^{i(p_1 x_1 - p'_1 x'_1)} \langle a_{p'_1 \sigma'_1}^+, a_{p_1 \sigma_1} \rangle dp_1 dp'_1 = \\ &= \frac{1}{(2\pi)^3} \iint e^{i(p_1 x_1 - p'_1 x'_1)} F(p_1, \sigma_1, p'_1 \sigma'_1) dp_1 dp'_1. \end{aligned}$$

The equilibrium value is

$$F^0(p_1, p'_1) = n_{p_1} \delta(p_1 - p'_1) \delta(\sigma_1 - \sigma'_1), \quad n_p = 2/(e^{(\varepsilon - \lambda)/\theta} + 1),$$

$$\varepsilon = \hbar^2 p^2 / 2m, \quad D_1^0(x_1, x'_1) = \frac{1}{(2\pi)^3} \frac{\delta(\sigma_1 - \sigma'_1)}{2} \int e^{ip_1(x_1 - x'_1)} n_{p_1} dp_1.$$

The functions D , in contrast to Bogolyubov distribution functions ⁽¹⁾, are normalized to n , and not to 1. Let us take the kinetic equation with a self-consistent term in the Hartree approximation

$$i\hbar \frac{\partial D_1(t, x_1, x'_1)}{\partial t} + \frac{\hbar^2}{2m} (\Delta_{r_1} - \Delta_{r'_1}) D_1(t, x_1, x'_1) - \\ - D_1(t, x_1, x'_1) \int [\Phi(r_1 - r'_2) - \Phi(r'_1 - r'_2)] D_1(t, x'_2, x'_2) dx'_2 = I_{st}. \quad (1)$$

The case $I_{st} = 0$ was considered in work ⁽²⁾. We linearize equation (1) and perform a symmetric Fourier transformation with respect to spatial variables of the type

$$F\{\dots\} = \frac{1}{(2\pi)^3} \int \{\dots\} e^{-ikr + ik'r'} dr dr'. \quad (2)$$

As a result we obtain the equation in k -space

$$\left[i\hbar \frac{\partial}{\partial t} - \frac{\hbar^2}{2m} (k_1^2 - k'_1{}^2) \right] \delta F(k_1 k'_1) - \\ - \frac{\Delta(\sigma_1 - \sigma'_1)}{2(2\pi)^{3/2}} \rho(k_1 - k'_1) \Phi(k_1 - k'_1) (n_{k'_1} - n_{k_1}) = I_{st}^{lin} \quad (3)$$

where

$$\rho(q) = \sum_{\sigma_2} \int \delta F(k, \sigma_2, k - q, \sigma_2) dk; \quad p_1 - p'_1 = q, \quad p_1 = k.$$

Let us imagine that the system is placed in an infinitesimally small external field and that the variation δF is due precisely to this field. Let

$$\delta H = e^{-iEt} \delta \Omega(x_1, x'_2) / 2\pi,$$

then (3) becomes

$$i\hbar \frac{\partial \delta F(t, k_1, \sigma_1, k'_1, \sigma'_1)}{\partial t} - \frac{\hbar^2}{2m} (k_1^2 - k'_1{}^2) \delta F(t, k_1, \sigma_1, k'_1, \sigma'_1) - \\ - \frac{\delta \Omega(t, k_1, \sigma_1, k'_1, \sigma'_1)}{4\pi} (n_{k'_1} - n_{k_1}) -$$

$$-\frac{\Delta(\sigma_1 - \sigma'_1)}{2(2\pi)^{3/2}} \rho(k_1 - k'_1) \Phi(k_1 - k'_1) (n_{k'_1} - n_{k_1}) = I_{\text{st}}^{\text{lin}}. \quad (4)$$

We apply the theorem on variations of the mean value of a dynamical quantity (see ⁽³⁾), and for the classical case ^(2,4))

$$\delta F(t, k_1, k'_1, \sigma_1, \sigma'_1) / \delta \Omega(t, p, p', \sigma, \sigma') = G^{\text{ret}}(E; k_1, k'_1, \sigma_1, \sigma'_1; p, p', \sigma, \sigma'), \quad (5)$$

$$\delta \Omega(t, k_1, k'_1, \sigma_1, \sigma'_1) / \delta \Omega(t, p, p', \sigma, \sigma') = \delta(k_1 - p) \delta(k'_1 - p') \Delta(\sigma - \sigma_1) \Delta(\sigma'_1 - \sigma'). \quad (6)$$

Then for the Green's function we obtain the equation

$$\begin{aligned} & \hbar E G_E(k_1, k'_1, \sigma_1, \sigma'_1; k, k', \sigma, \sigma') - \frac{\hbar^2}{2m} (k_1^2 - k'^2) G_E - \\ & - \frac{\Delta(\sigma_1 - \sigma'_1)}{2(2\pi)^{3/2}} \Phi(k_1 - k'_1) \rho(k_1 - k'_1; k, k', \sigma, \sigma') (n_{k'_1} - n_{k_1}) - \\ & - \frac{\Delta(\sigma - \sigma_1) \Delta(\sigma' - \sigma'_1)}{4\pi} \delta(k - k_1) \delta(k' - k'_1) (n_{k'_1} - n_{k_1}) = I_{\text{st}}^G, \\ & \rho(q; k, k', \sigma, \sigma') = \sum_{\sigma_1} \int G_E(k_1, k_1 - q, \sigma_1, \sigma_1; k, k', \sigma, \sigma') dk_1. \end{aligned} \quad (7)$$

It remains to choose the collision integral and to write it in terms of Green's functions. The quantum collision integral was obtained in works ^(5,6), etc. Let us consider the expression obtained by Wyld and Fried ⁽⁷⁾ for the spatially homogeneous case and for particles without spin,

$$\begin{aligned} I_{\text{st}} = & -\frac{i}{(2\pi)^2} \iint dp' dq' \left| \frac{\Phi(q')}{K \{q', \frac{1}{\hbar}(E_{p+q'} - E_p)\}} \right|^2 \times \\ & \times \delta(E_{p+q'} + E_{p'-q'} - E_p - E_{p'}) \{F(p)F(p')[1 - F(p + q')]\} \times \\ & \times [1 - F(p' - q')] - F(p + q')F(p' - q')[1 - F(p)][1 - F(p')], \end{aligned} \quad (8)$$

where

$$K(q, \Omega) = 1 + \frac{\Phi(q)}{(2\pi)^{3/2}} \int \frac{F(p'' + q/2) - F(p'' - q/2)}{\hbar\Omega + E_{p''-q/2} - E_{p''+q/2} + i\eta} dp'', \quad (9)$$

$$E_p = \hbar^2 p^2 / 2m, \quad \eta > 0, \quad \eta \rightarrow 0.$$

Let us recall that this integral was obtained under the assumption that the higher distribution functions depend on time only through the first func-

i.e., the equation is valid for $t > \tau_{\text{coll}}$. Moreover, I_{st} is proportional to the small parameter $\varepsilon = 8\pi e^2 m / r_d p_F^2$,

$$r_d = (8\pi e^2 m n / p_F^2)^{-1/2}, \quad n = N/V, \quad p_F = (3\pi^2 n)^{1/3} \hbar, \\ \varepsilon = \sqrt{\pi} / 3 (8e^2 m / \hbar p_F)^{3/2} = \sqrt{\pi} / 3 \sigma^{3/2} \ll 1. \quad (10)$$

Consequently, in the present theory ε , or $\sigma = 8e^2 m / p_F \hbar$, are small parameters. Since $\varepsilon = (8\pi e^2 m / \hbar^2)^{3/2} / 3\pi^2 n^{1/2}$, it is clear that $\varepsilon \ll 1$ for sufficiently dense systems at low temperatures. Let us calculate the collision integral (8) and assume that the linearized equation obtained is also valid for a spatially inhomogeneous system.

Applying the variational theorem to (5), (6), we find

$$I_{\text{st}}^G = -\frac{i}{(2\pi)^2} \iint dp' dq Q^0(q, k_1) \delta(E_{k_1+q} + E_{p'-q} - E_{k_1} - E_{p'}) \times \\ \times \{G_E(k_1, k'_1, \sigma_1, \sigma'_1; k, k', \sigma, \sigma') s_1 + G_E(p', k'_1, \sigma_1, \sigma'_1; k, k', \sigma, \sigma') s_2 - \\ - G_E(k_1 + q, k'_1, \sigma_1, \sigma'_1; k, k', \sigma, \sigma') s_3 - \\ - G_E(p' - q, k'_1, \sigma_1, \sigma'_1; k, k', \sigma, \sigma') s_4\}, \quad (11)$$

where

$$Q^0(q, k_1) = \left| \frac{\Phi(q)}{K^0(q, (1/\hbar)(E_{k_1+q} - E_{k_1}))} \right|^2;$$

$$s_1 = n_{p'} - n_{p'} n_{k_1+q} - n_{p'} n_{p'-q} + n_{k_1+q} n_{p'-q}; \\ s_2 = n_{k_1} - n_{k_1} n_{k_1+q} - n_{k_1} n_{p'-q} + n_{k_1+q} n_{p'-q}; \\ s_3 = n_{p'-q} + n_{k_1} n_{p'} - n_{p'-q} n_{k_1} - n_{p'-q} n_{p'}; \\ s_4 = n_{k_1+q} + n_{k_1} n_{p'} - n_{k_1+q} n_{k_1} - n_{k_1+q} n_{p'},$$

and K^0 means that n_p has been substituted in place of F .

The solution of equation (7), which for simplicity we consider for spinless particles, with I_{st}^G in the form (11), will symbolically be

$$G = \frac{L + \mathcal{L}}{\hbar E - (\hbar^2/2m)(k_1^2 - k_1'^2) + iM}, \quad (12)$$

where

$$\begin{aligned} M &= \frac{1}{(2\pi)^2} \iint dp' dq Q^0(q, k_1) s_1 \delta(E_{k_1+q} + E_{p'-q} - F_{k_1} - E_{p'}), \\ L &= \frac{1}{(2\pi)^{3/2}} \rho(k_1 - k_1'; k, k') \Phi(k_1 - k_1') (n_{k_1'} - n_{k_1}) + \\ &\quad + \frac{1}{2\pi} \delta(k - k_1) \delta(k_1' - k') (n_{k_1'} - n_{k_1}), \\ \mathcal{L} &= I_{st}^G - iMG_E. \end{aligned} \quad (13)$$

The structure of expression (12) is analogous to the classical expression for the Green's function. Therefore here, too, one may neglect $\mathcal{L}$ with high accuracy for values of the wave numbers much smaller than k_F . Let us show this by passing to the dimensionless k/k_F and q/k_F . $L \sim e^2/q^2$. Q^0 will be $\sim me^4/k_F^3$. In order not to take $\mathcal{L}$ into account, the condition $\mathcal{L} \ll L$ must be fulfilled; hence $q^2 \ll k_F^2/\sigma$, i.e., for $q < k_F$ and even $q \sim k_F$, since $\sigma \ll 1$.

Therefore, for $q < k_F$ we have

$$G_E = L/[\hbar E - (\hbar^2/2m)(k_1^2 - k_1'^2) + iM]. \quad (14)$$

Introduce the substitution $k_1' + q = k_1$, $k_1 = k_1$. Then (14) becomes

$$\begin{aligned} &[\hbar E - (\hbar^2/2m)(2k_1 - q)q + iM(k_1)] G_E(k_1, k_1 - q; k, k') - \\ &\quad - (2\pi)^{-3/2} \rho(q; k, k') \Phi(q) (n_{k_1-q} - n_{k_1}) = \\ &= (2\pi)^{-1} \delta(k - k_1) \delta(k' + q - k_1) (n_{k_1-q} - n_{k_1}). \end{aligned} \quad (15)$$

Next, after simple operations of division and integration, we arrive at the following solution for the Green's functions:

$$\rho(q; k, k') = \frac{1}{2\pi} \frac{(n_{k'} - n_k) \delta(k' + q - k)}{\hbar E - (\hbar^2/2m)(2k - q)q + iM(k)} \times \left[1 + \frac{\Phi(q)}{(2\pi)^{3/2}} \int \frac{(n_k - n_{k-q}) dk}{\hbar E - (\hbar^2/2m)(2k - q)q + iM(k)} \right]^{-1}. \quad (16)$$

Thus, in contrast to work ⁽²⁾, here the damping of the system due to collisions is taken into account. Damping without allowance for polarization by the method of kinetic equations was considered in work ⁽⁸⁾.

Knowing the function ρ , with the aid of the spectral theorems ⁽³⁾ it is not difficult to calculate the thermodynamic characteristics of the system, as well as the correlation functions. For this it is necessary to find the spectral density I from the formula

$$I(E) = -2 \operatorname{Im} G_E / (e^{E/\theta} + 1).$$

After simple calculations we obtain

$$\begin{aligned} I(E) = & -\delta(k - k' - q)(n_{k'} - n_k) \{M(k) \operatorname{Re} K^0 + \\ & + [\hbar E - (\hbar^2/2m)(2k - q)q] \operatorname{Im} K^0\} \times \\ & \times \{2\pi(e^{E/\theta} + 1)(\operatorname{Re}^2 K^0 + \operatorname{Im}^2 K^0)[[\hbar E - \\ & - (\hbar^2/2m)(2k - q)q]^2 + M^2(k)]\}^{-1}. \end{aligned}$$

The damping and frequencies of the system are determined as the poles of the Green's function (16). For this purpose, the values K^0 and M are first calculated from formulas (9) and (13). In the case of small values of the wave numbers at $\theta = 0$,

$$Q^0(q) = \frac{2e^4}{\pi(q^2 + 4ck_F)^2}, \quad c = \frac{me^4}{2\pi\hbar^2} \sim \sigma.$$

For $\theta = 0$, and also for small values of the wave numbers, the frequencies coincide with the results of the works of V. P. Silin; the collisional damping is absent for $k \ll k_F$. It appears only in a term $\sim (k/k_F)^2$, which corresponds to the result of Dubois ⁽⁹⁾.

Analogous results are also obtained for $\theta \ll \varepsilon_F = \mu_0$, with the values $Q^0(q)$ differing from the case $\theta = 0$ only by a change of the factor

$$c^* \rightarrow c : \quad c^* = c \left\{ 1 - \frac{\pi^2}{12} \left(\frac{\theta}{\mu_0} \right)^2 \right\}.$$

It should be noted that the exact limits of applicability of equation (11) can be estimated only on the basis of a more general inhomogeneous quantum kinetic equation with collisions. For the present one may say that it is applicable for times in the interval $\tau_{\text{int}} < t < \tau_{\text{rel}}$, where τ_{rel} is determined by the damping of the system, and also in the interval of wave vectors $q < k_F$.

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Moscow State University
named after M. V. Lomonosov

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