

ON THE TOPOLOGICAL STRUCTURE OF SPACE IN RELATIVISTIC MECHANICS

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Abstract

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MATHEMATICS

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ON THE TOPOLOGICAL STRUCTURE OF SPACE IN RELATIVISTIC MECHANICS

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I. In a Riemannian space R_n with positive definite metric form, a topology may be introduced in several equivalent ways. In particular, neighborhoods $O(x)$ may be specified either from the metric $[O(x, \varepsilon) = \{y : \rho(x, y) < \varepsilon\}]$, or from the coordinates $(O(x, \varepsilon) = \{y : |y_i - x_i| < \varepsilon, i = 1, \dots, n\})$.

In the theory of relativity, the choice of neighborhoods by metric, by the modulus of the metric, or by coordinates leads (because of the indefiniteness of the quadratic form) to different topologies. The question arises of the physical justification for choosing one or another system of neighborhoods. Under any of the indicated definitions of neighborhoods in pseudo-Euclidean space, the induced intrinsic topology of timelike lines corresponding to the trajectories of real particles is the same and coincides with the topology of trajectories in nonrelativistic mechanics. The intrinsic topology of trajectories therefore appears to be something more fundamental in comparison with the topology of space as a whole. In experiment one always considers some concrete physical system and studies its trajectories. The geometry of the entire space, which does not admit direct investigation, must be constructed from the intrinsic geometry of trajectories, taking their intersections into account.

II. Let some system of subsets $\{M_\alpha\}$ be given in a set M , and let some topology be defined in each M_α .

The **topological union** of the family $\{M_\alpha\}$ will mean the set

$$N = \bigcup_{\alpha} M_{\alpha}$$

with the following topology: $A \subseteq N$ is open in N if and only if $A \cap M_\alpha$ is open in each M_α . We shall call the topological union **exact** if each M_α is a subspace of N , i.e., if every set open in M_α is the intersection with M_α of some set open in N .

It is natural to construct the space from trajectories in the form of their topological union.

III. Complexes of $n + 1$ numbers will be interpreted as coordinates $(x_0, x_1, \dots, x_n)$ of a point x . The set defined by the relation

$$\sum_{i=1}^n (y_i - x_i)^2 - (y_0 - x_0)^2 = 0$$

is the light cone ($c = 1$) with vertex at x . A basis at the point x is formed by the sets $O(x, \varepsilon, \eta)$ of points for which

$$\sum_{i=1}^n (y_i - x_i)^2 / (y_0 - x_0)^2 < 1 + \eta; \quad \sum_{i=0}^n (y_i - x_i)^2 < \varepsilon;$$

by definition we put $x \in O(x, \varepsilon, \eta)$. For $\eta = 0$, the indicated construction gives the exact topological union of timelike world

* The intersection pattern of the family $\{M_\alpha\}$ is essential. For example, the topological union of two unit intervals in which, say, points with rational coordinate are considered common is not even a Hausdorff space.

lines. The structure of the space Π_n , which is the topological union of all trajectories (including lightlike ones), is fairly complex. The space $\tilde{\Pi}_n$ constructed above, having fewer open sets, is a simple model that preserves the essential features of the more complicated topology of Π_n .

The following assertions characterize the topological structure of $\tilde{\Pi}_n$.

1. $\tilde{\Pi}_n$ is a completely regular nonnormal (and hence nonmetrizable) space. Complete regularity is proved by applying the criterion of Yu. M. Smirnov⁽¹⁾. To verify the absence of normality it suffices to consider, for example, the sets with rational and irrational coordinates on a spacelike segment. Both of them are closed in the topology Π_n .
2. $\tilde{\Pi}_n$ has no countably paracompact (and, a fortiori, no countable) base. Indeed, a regular space with a countably paracompact base would be normal⁽²⁾. $\tilde{\Pi}_n$, however, satisfies the first axiom of countability at all points.
3. $\tilde{\Pi}_n$ is neither locally compact nor paracompact.
4. $\text{ind } \tilde{\Pi}_n = 1$; $\dim \tilde{\Pi}_n = n$ for $n \leq 2$. Apparently, $\dim \tilde{\Pi}_n = n$ for all n .
5. Spacelike "lines" in Π_n have dimension zero and are totally disconnected. The dimension of timelike lines is 1.
6. All functions continuous in the topology R_{n+1} are continuous also in $\tilde{\Pi}_n$. However, the class of continuous functions in $\tilde{\Pi}_n$ is broader. As an example, we indicate the function, continuous in $\tilde{\Pi}_1$, $f(x, t) = \exp[-t/(x^2 + t^2)] \times \sin[1/(x^2 + t^2)]$.

IV. In the preceding section, the points of the set in which the topology had to be introduced were fixed in advance by means of a certain coordinate system. The world lines constituting the initial family for forming the topological union were defined by equations in these coordinates.

Suppose now that we have a family of sets $\{M_\alpha\}$ such that each M_α is homeomorphic to a simple arc, and the set of indices α has the cardinality of the continuum. Next let us prescribe some intersection scheme for the family M_α , i.e., indicate, for each pair of sets $M_{\alpha'}, M_{\alpha''}$, the points that we regard as common to them. Under what intersection scheme is the topological union $\{M_\alpha\}$ homeomorphic to Π_n ? Of course, for different n the intersection schemes are, generally speaking, not the same. Which of them leads to the space Π_3 realizable in mechanics?

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CITED LITERATURE

¹ Yu. M. Smirnov, DAN, 62, No. 6 (1948). ² Yu. M. Smirnov, DAN, 77, No. 2 (1951).

Note: Figure translations are in progress. See original paper for figures.

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