

ON COMPLETELY IRREDUCIBLE REPRESENTATIONS OF $\backslash(\mathrm{SU}(2,1)\backslash)$

A. I. Shtern

1968

SovietRxiv

View the original and related papers at <https://sovietsrxiv.org/items/ru-196801.19832>

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

Abstract

Full Text

UDC 519.46

MATHEMATICS

A. I. SHTERN

ON COMPLETELY IRREDUCIBLE REPRESENTATIONS OF $SU(2, 1)$

(Presented by Academician L. S. Pontryagin on May 6, 1967)

Let G be a real semisimple Lie group and let $G = UAN$ be its Iwasawa decomposition (see, for example, (7)). Let M be the centralizer of A in G , $MN = \{\mu n, \mu \in M, n \in N\}$. We consider representations of the group G induced by finite-dimensional irreducible representations of MN under which N goes to the identity. The paper describes the structure of these representations for the group $G = SU(2, 1)$ of complex unimodular matrices of order three preserving the indefinite Hermitian form $|z_1|^2 + |z_2|^2 - |z_3|^2$, and finds conditions for equivalence and unitarity of the representations. Representations of this group were considered in (2-5).

1. Let G' be the group, isomorphic to G , of complex unimodular matrices of order three preserving the form $z_1 \bar{z}_3 + z_2 \bar{z}_2 + z_3 \bar{z}_1$. Then

$$G = \left\{ sg' s^{-1} \mid g' \in G', s = s^{-1} = \begin{pmatrix} 1/\sqrt{2} & 0 & 1/\sqrt{2} \\ 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -1/\sqrt{2} \end{pmatrix} \right\}. \quad (1)$$

Consider the subgroups K, U, Γ, M, N of the group G :

$$K = \{k\} = \left\{ s \begin{pmatrix} \lambda^{-1} e^{i\varphi} & \lambda^{-1} e^{i\varphi} z & \lambda^{-1} e^{i\varphi} (-|z|^2/2 + it) \\ 0 & e^{-2i\varphi} & -e^{2i\varphi} \bar{z} \\ 0 & 0 & \lambda e^{i\varphi} \end{pmatrix} s \right\},$$

$$U = \{u\} = \left\{ \begin{pmatrix} u_{11} & u_{12} & 0 \\ u_{21} & u_{22} & 0 \\ 0 & 0 & u_{33} \end{pmatrix} \right\}, \quad \Gamma = \left\{ \gamma_\varphi : \begin{pmatrix} e^{i\varphi} & 0 & 0 \\ 0 & e^{-2i\varphi} & 0 \\ 0 & 0 & e^{i\varphi} \end{pmatrix} \right\}, \quad (2)$$

$$M = \{\mu \mid \mu \in K, z = 0, t = 0\}, \quad N = \{n \mid n \in K, \lambda = 1, \varphi = 0\},$$

$$K = MN, \quad \Gamma = M \cap N.$$

Here $\lambda > 0$, z is complex, t is real, $u^*u = 1$. Let $g = (a_{jk})_{j,k=1}^3$, $\beta(g) = a_{33} - a_{13}$, $\alpha(g) = |\beta(g)|^{i\sigma-m-2}\beta(g)^m$ (σ complex, m an integer). Let $\bar{u}\bar{g}$ be an element of U such that $u\bar{g} = k \cdot ug$ for some $k \in K$. Then the representation of the group G , induced by the representation of the subgroup $K = MN$: $k \rightarrow |k_{33}|^{i\sigma-m}k_{33}^m$, acts in the Hilbert space $\mathcal{L}_2^m(U) = \{f(u) \mid f(u) \in \mathcal{L}_2(U), f(\gamma_\varphi u) = e^{im\varphi}f(u) \text{ for } \gamma_\varphi \in \Gamma\}$ by the formula

$$[T^{\sigma,m}(g)f](u) = \frac{\alpha(ug)}{\alpha(\bar{u}\bar{g})} f(\bar{u}\bar{g}). \quad (3)$$

The operators $T^{\sigma,m}$ are bounded. The representation conjugate to $T^{\sigma,m}$ is $T^{\bar{\sigma},m}$.

Along with representations in $\mathcal{L}_2^m(U)$, it is convenient to consider representations given by formula (3) in the nuclear space $C_m^\infty(U)$ of infinitely differentiable functions on U satisfying the condition $f(\gamma_\varphi u) = e^{im\varphi}f(u)$, $\gamma_\varphi \in \Gamma$.

2. On invariant subspaces of the representations $T^{\sigma,m}$

The representations $\tilde{T}^{\sigma,m}$ in $\mathcal{L}_2^m(U)$ are completely irreducible if: 1) $i\sigma + m$ is not an even number, or 2) $\sigma = m = 0$. If $(i\sigma + m)/2 = p$, where p is an integer, then let us introduce the integer $q = (i\sigma - m)/2$ and denote the representation $T^{(p+q)/i, p-q}$ by $T_{p,q}$.

In the decomposition of the restriction of the representation $T^{\sigma,m}$ to the subgroup U , there enter, with multiplicity one, the irreducible representations of U (isomorphic to $U(2)$) with weights $f_1 = 2\alpha - \beta - m$, $f_2 = \alpha - 2\beta - m$, where $\alpha, \beta \geq 0$ are integers. These representations are realized in subspaces $\mathfrak{M}^{\alpha,\beta}$ spanned by functions from $\mathcal{L}_2^m(\bar{U})$ which are matrix elements of the corresponding irreducible representations: $C_{(\beta-\alpha)/2,j}^{f_1,f_2}$, $|j| \leq (\alpha + \beta)/2$. The invariant subspaces of the representations $T_{p,q}$ are direct sums of the subspaces $\mathfrak{M}^{\alpha,\beta}$. Introduce the following notation:

$$\begin{aligned} \mathfrak{M}_1^{p,q} &= \sum_{\substack{0 \leq \alpha < p \\ 0 \leq \beta < +\infty}} \oplus \mathfrak{M}^{\alpha,\beta}; & \mathfrak{M}_2^{p,q} &= \sum_{\substack{0 \leq \alpha < +\infty \\ 0 \leq \beta < q}} \oplus \mathfrak{M}^{\alpha,\beta}; \\ \mathfrak{M}_{p,q}^1 &= \sum_{\substack{-q < \alpha < +\infty \\ 0 \leq \beta < +\infty}} \oplus \mathfrak{M}^{\alpha,\beta}; & \mathfrak{M}_{p,q}^2 &= \sum_{\substack{0 \leq \alpha < +\infty \\ -p < \beta < +\infty}} \oplus \mathfrak{M}^{\alpha,\beta}. \end{aligned}$$

Of course, these subspaces may turn out to be trivial. The structure of the representation $T_{p,q}$ depends on the signs of the numbers $p, q, p + q$.

- a) $p \geq 1, q \geq 1$. The subspaces $\mathfrak{M}_1^{p,q}; \mathfrak{M}_2^{p,q}; \mathfrak{M}_1^{p,q} + \mathfrak{M}_2^{p,q}; \mathfrak{M}_1^{p,q} \cap \mathfrak{M}_2^{p,q}$ are invariant. The representations induced by $T_{p,q}$ in $\mathfrak{M}_1^{p,q} \cap \mathfrak{M}_2^{p,q}; \mathfrak{M}_1^{p,q} / \mathfrak{M}_1^{p,q} \cap \mathfrak{M}_2^{p,q}; \mathfrak{M}_2^{p,q} / \mathfrak{M}_1^{p,q} \cap \mathfrak{M}_2^{p,q}; \mathcal{L}_2^{-q}(U) / \mathfrak{M}_1^{p,q} + \mathfrak{M}_2^{p,q}$ are completely irreducible.

- b) $p \leq -1, q \leq -1$. The subspaces $\mathfrak{M}_{p,q}^1; \mathfrak{M}_{p,q}^2; \mathfrak{M}_{p,q}^1 + \mathfrak{M}_{p,q}^2; \mathfrak{M}_{p,q}^1 \cap \mathfrak{M}_{p,q}^2$ are invariant. The representations induced by $T_{p,q}$ in $\mathfrak{M}_{p,q}^1 \cap \mathfrak{M}_{p,q}^2; \mathfrak{M}_{p,q}^1/\mathfrak{M}_{p,q}^1 \cap \mathfrak{M}_{p,q}^2; \mathfrak{M}_{p,q}^2/\mathfrak{M}_{p,q}^1 \cap \mathfrak{M}_{p,q}^2; \mathcal{L}_2^{p-q}(U)/\mathfrak{M}_{p,q}^1 + \mathfrak{M}_{p,q}^2$ are completely irreducible.
- c) $p \geq 1, q \leq -1, p+q \geq 1$. $\mathfrak{M}_1^{p,q}; \mathfrak{M}_{p,q}^1; \mathfrak{M}_1^{p,q} \cap \mathfrak{M}_{p,q}^1$ are invariant. The representations in $\mathfrak{M}_1^{p,q} \cap \mathfrak{M}_{p,q}^1; \mathfrak{M}_1^{p,q}/\mathfrak{M}_1^{p,q} \cap \mathfrak{M}_{p,q}^1; \mathfrak{M}_{p,q}^1/\mathfrak{M}_1^{p,q} \cap \mathfrak{M}_{p,q}^1$ are completely irreducible.
- d) $p \geq 1, q \leq -1, p+q \leq 1$. $\mathfrak{M}_1^{p,q}; \mathfrak{M}_{p,q}^1; \mathfrak{M}_1^{p,q} \oplus \mathfrak{M}_{p,q}^1$ are invariant. The representations in $\mathfrak{M}_1^{p,q}; \mathfrak{M}_{p,q}^1; \mathcal{L}_2^{p-q}(U)/\mathfrak{M}_1^{p,q} \oplus \mathfrak{M}_{p,q}^1$ are completely irreducible.
- e) $p \leq -1, q \geq 1, p+q \geq 1$. $\mathfrak{M}_2^{p,q}; \mathfrak{M}_{p,q}^2; \mathfrak{M}_2^{p,q} \cap \mathfrak{M}_{p,q}^2$ are invariant. The representations in $\mathfrak{M}_2^{p,q} \cap \mathfrak{M}_{p,q}^2; \mathfrak{M}_2^{p,q}/\mathfrak{M}_2^{p,q} \cap \mathfrak{M}_{p,q}^2; \mathfrak{M}_{p,q}^2/\mathfrak{M}_2^{p,q} \cap \mathfrak{M}_{p,q}^2$ are completely irreducible.
- f) $p \leq -1, q \geq 1, p+q \leq -1$. $\mathfrak{M}_2^{p,q}; \mathfrak{M}_{p,q}^2; \mathfrak{M}_2^{p,q} \oplus \mathfrak{M}_{p,q}^2$ are invariant. The representations in $\mathfrak{M}_2^{p,q}; \mathfrak{M}_{p,q}^2; \mathcal{L}_2^{p-q}(U)/\mathfrak{M}_2^{p,q} \oplus \mathfrak{M}_{p,q}^2$ are completely irreducible.
- g) $p = 0, q \geq 1$. $\mathfrak{M}_2^{0,q}$ is invariant. The representations in $\mathfrak{M}_2^{0,q}$ and $\mathcal{L}_2^{-q}(U)/\mathfrak{M}_2^{0,q}$ are completely irreducible.
- h) $p = 0, q \leq -1$. $\mathfrak{M}_{0,q}^1$ is invariant. The representations in $\mathfrak{M}_{0,q}^1$ and $\mathcal{L}_2^{-q}(U)/\mathfrak{M}_{0,q}^1$ are completely irreducible.
- i) $p \geq 1, q = 0$. $\mathfrak{M}_1^{p,0}$ is invariant. The representations in $\mathfrak{M}_1^{p,0}$ and $\mathcal{L}_2^p(U)/\mathfrak{M}_1^{p,0}$ are completely irreducible.
- j) $p \leq -1, q = 0$. $\mathfrak{M}_{p,0}^2$ is invariant. The representations in $\mathfrak{M}_{p,0}^2$ and $\mathcal{L}_2^p(U)/\mathfrak{M}_{p,0}^2$ are completely irreducible.
- k) $p \geq 1, q \leq -1, p+q = 0$. $\mathfrak{M}_1^{p,q}; \mathfrak{M}_{p,q}^1$ are invariant, $\mathfrak{M}_1^{p,q} \oplus \mathfrak{M}_{p,q}^1 = \mathcal{L}_2^{p-q}(U)$. The representations in $\mathfrak{M}_1^{p,q}$ and $\mathfrak{M}_{p,q}^1$ are completely irreducible.
- l) $p \leq -1, q \geq 1, p+q = 0$. $\mathfrak{M}_2^{p,q}; \mathfrak{M}_{p,q}^2$ are invariant, $\mathfrak{M}_2^{p,q} \oplus \mathfrak{M}_{p,q}^2 = \mathcal{L}_2^{p-q}(U)$. The representations in $\mathfrak{M}_2^{p,q}$ and $\mathfrak{M}_{p,q}^2$ are completely irreducible.

3. On the equivalence* of representations. If $i\sigma + m$ is not an even number, then the representation $T^{\sigma,m}$ is not equivalent to any of the subrepresentations or factor-representations $T_{p,q}$.

If $i\sigma_1 + m_1$ and $i\sigma_2 + m_2$ are both not even numbers, then the representations T^{σ_1,m_1} and T^{σ_2,m_2} are equivalent if and only if $\sigma_1 = \sigma_2, m_1 = m_2$, or $\sigma_1 = -\sigma_2, m_1 = m_2$.

The representations $T_{p,q}$ are pairwise inequivalent, but the following equivalences occur:

- 1) The subrepresentation $T_{p,q}$ with $p \geq 1$, $q \geq 1$ in the subspace $\mathfrak{M}_1^{p,q} \cap \mathfrak{M}_2^{p,q}$ is equivalent to the factor-representation $T_{-q,-p}$ in $\mathcal{L}_2^{p-q}(u)/\mathfrak{M}_{-q,-p}^1 + \mathfrak{M}_{-q,-p}^2$.
- 2) The subrepresentation $T_{p,q}$ with $p \geq 1$, $q \leq -1$, $p+q \leq -1$ in the subspace $\mathfrak{M}_1^{p,q}$ is equivalent to the factor-representation $T_{-q,-p}$ in $\mathcal{L}_2^{p-q}(u)/\mathfrak{M}_{-q,-p}^1 = \mathfrak{M}_1^{-q,-p}/\mathfrak{M}_1^{-q,-p} \cap \mathfrak{M}_{-q,-p}^1$.
- 3) The subrepresentation $T_{p,q}$ with $p \leq -1$, $q \geq 1$, $p+q \leq -1$ in the subspace $\mathfrak{M}_2^{p,q}$ is equivalent to the factor-representation $T_{-q,-p}$ in $\mathcal{L}_2^{p-q}(u)/\mathfrak{M}_{-q,-p}^2 = \mathfrak{M}_2^{-q,-p}/\mathfrak{M}_2^{-q,-p} \cap \mathfrak{M}_{-q,-p}^2$.
- 4) The subrepresentation $T_{0,q}$ with $q \geq 1$ in $\mathfrak{M}_2^{0,q}$ is equivalent to the factor-representation $T_{-q,0}$ in $\mathcal{L}_2^{-q}(u)/\mathfrak{M}_{-q,0}^2$.
- 5) The subrepresentation $T_{p,0}$ with $p \geq 1$ in $\mathfrak{M}_1^{p,0}$ is equivalent to the factor-representation $T_{0,-p}$ in $\mathcal{L}_2^p(u)/\mathfrak{M}_{0,-p}^1$.
- 6) The factor-representation $T_{p,q}$ with $p \geq 1$, $q \geq 1$ in the factor-space

$$\mathfrak{M}_1^{p,q}/\mathfrak{M}_1^{p,q} \cap \mathfrak{M}_2^{p,q}$$

is equivalent to the factor-representation $T_{-q,-p}$ in

$$\mathfrak{M}_{-q,-p}^2/\mathfrak{M}_{-q,-p}^1 \cap \mathfrak{M}_{-q,-p}^2,$$

to the subrepresentation $T_{p+q,-q}$ in $\mathfrak{M}_{p+q,-q}^1 \cap \mathfrak{M}_{p+q,-q}^1$, and to the factor-representation $T_{q,-p-q}$ in

$$\mathcal{L}_2^{p+2q}(u)/\mathfrak{M}_1^{q,-p-q} \oplus \mathfrak{M}_{q,-p-q}^1.$$

- 7) The factor-representation $T_{p,q}$ with $p \geq 1$, $q \geq 1$ in

$$\mathfrak{M}_2^{p,q}/\mathfrak{M}_1^{p,q} \cap \mathfrak{M}_2^{p,q}$$

is equivalent to the factor-representation $T_{-q,-p}$ in

$$\mathfrak{M}_{-q,-p}^1/\mathfrak{M}_{-q,-p}^1 \cap \mathfrak{M}_{-q,-p}^2,$$

to the subrepresentation $T_{-p,p+q}$ in $\mathfrak{M}_{-p,p+q}^2 \cap \mathfrak{M}_{-p,p+q}^2$, and to the factor-representation $T_{-p-q,p}$ in

$$\mathcal{L}_2^{-2p-q}(u)/\mathfrak{M}_2^{-p-q,p} \oplus \mathfrak{M}_{-p-q,p}^2.$$

- 8) The factor-representation $T_{p,q}$ with $p \geq 1$, $q \geq 1$ in

$$\mathcal{L}_2^{p-q}(u)/\mathfrak{M}_1^{p,q} + \mathfrak{M}_2^{p,q}$$

is equivalent to the subrepresentation $T_{-q,-p}$ in $\mathfrak{M}_{-q,-p}^1 \cap \mathfrak{M}_{-q,-p}^2$, to the subrepresentation $T_{q,-p-q}$ in $\mathfrak{M}_{q,-p-q}^1$, to the subrepresentation $T_{-p-q,p}$ in $\mathfrak{M}_{-p-q,p}^2$, to the factor-representation $T_{p+q,-q}$ in

$$\mathcal{L}_2^{p+2q}(u)/\mathfrak{M}_1^{p+q,-q} = \mathfrak{M}_1^{p+q,-q}/\mathfrak{M}_1^{p+q,-q} \cap \mathfrak{M}_1^{p+q,-q}$$

and to the factor-representation $T_{-p,p+q}$ in

$$\mathcal{L}_2^{-q-2p}(u)/\mathfrak{M}_2^{-p,p+q} = \mathfrak{M}_2^{-p,p+q}/\mathfrak{M}_2^{-p,p+q} \cap \mathfrak{M}_2^{-p,p+q}.$$

- 9) The subrepresentation $T_{0,-q}$ with $q \leq -1$ in $\mathfrak{M}_{0,q}^1$ is equivalent to the factor-representation $T_{-q,0}$ in $\mathcal{L}_2^{-q}(u)/\mathfrak{M}_1^{-q,0}$ and to the subrepresentation $T_{q,-q}$ in $\mathfrak{M}_{q,-q}^2$.
- 10) The subrepresentation $T_{p,0}$ with $p \leq -1$ in $\mathfrak{M}_{p,0}^2$ is equivalent to the factor-representation $T_{0,-p}$ in $\mathcal{L}_2^p(u)/\mathfrak{M}_2^{0,-p}$ and to the subrepresentation $T_{p,-p}$ in $\mathfrak{M}_{-p,p}^1$.

In all other cases the representations are pairwise inequivalent.

Thus, in each set of pairwise equivalent representations there is at least one representation in an irreducible invariant subspace $\mathcal{L}_2^{p-q}(u)$ and at least one representation in the factor-space of the space of the representation $T_{p,q}$ by a maximal invariant subspace.

4. **On the unitarity of the representations $T^{\sigma,m}$ and the subrepresentations $T_{p,q}$.** In $C_m^\infty(u)$ one can introduce a nonzero Hermitian functional invariant with respect to $T^{\sigma,m}$ only for: 1) real σ , 2) purely imaginary σ . In the first case the functional is definite and has the form

$$B(\varphi, \psi) = \int_U \varphi(u) \overline{\psi(u)} du,$$

where du is Haar measure on U . In the second case

* Here and below, in the sense of (1).

the functional is definite for $m = 0$, $-2 < i\sigma < 2$ and $-1 < i\sigma < 1$, $m = 2k + 1$, k an integer. At the other nonintegral points the functional for purely imaginary σ gives an indefinite scalar product with an infinite number of positive and negative squares.

For all subrepresentations $T_{p,q}$ there exist invariant continuous Hermitian functionals. They are definite in the following cases.

The subrepresentation $T_{1,1}$ in the one-dimensional subspace $\mathfrak{M}_1^{1,1} \cap \mathfrak{M}_2^{1,1}$.

The subrepresentations $T_{p,q}$ with $p \geq 1$, $q \leq -1$, $p + q \leq -1$ in the subspace $\mathfrak{M}_1^{p,q}$.

The subrepresentations $T_{p,q}$ with $p \leq -1$, $q \geq 1$, $p + q \leq -1$ in the subspace $\mathfrak{M}_2^{p,q}$.

The subrepresentation $T_{0,1}$ in $\mathfrak{M}_2^{0,1}$.

The subrepresentation $T_{1,0}$ in $\mathfrak{M}_1^{1,0}$.

The subrepresentations $T_{p,-p+1}$ for $p \geq 2$ in $\mathfrak{M}_1^{p,-p+1} \cap \mathfrak{M}_{p,-p+1}^1$.

The subrepresentations $T_{1-q,q}$ for $q \geq 2$ in $\mathfrak{M}_2^{1-q,q} \cap \mathfrak{M}_{1-q,q}^2$.

The subrepresentations $T_{p,q}$ with $p \leq -1$, $q \leq -1$ in $\mathfrak{M}_{p,q}^1 \cap \mathfrak{M}_{p,q}^2$.

The subrepresentations $T_{p,-p}$ for $p \leq -1$ in $\mathfrak{M}_{p,-p}^2$ and $\mathfrak{M}_{p,-p}^2$.

The subrepresentations $T_{p,-p}$ for $p \geq 1$ in $\mathfrak{M}_1^{p,-p}$ and $\mathfrak{M}_{p,-p}^1$.

Invariant Hermitian functionals for representations not equivalent to those listed above are indefinite and, for infinite-dimensional representations, contain an infinite number of both positive and negative squares.

5. Using the result of Harish-Chandra ((6), Theorem 4), we obtain:

Theorem. *Every completely irreducible representation of the group $SU(2,1)$ in a Banach space is equivalent (in the sense of (1)) to one of the representations described in Section 2. Every irreducible unitary representation of $SU(2,1)$ is unitarily equivalent to one of the representations indicated in Section 4.*

6. Unitary representations of the group $SU(2,1)$ were studied in papers (2-5). In (2,4) the representations $T^{\sigma,m}$ are described for real σ and the subrepresentations $T_{p,q}$ for the cases $p \geq 1$, $q \leq -1$, $p + q \leq -1$ and $p \geq 1$, $p + q = 0$ in the subspace $\mathfrak{M}_1^{p,q}$, and for the cases $p \leq -1$, $q \geq 1$, $p + q \leq -1$ and $q \geq 1$, $p + q = 0$ in the subspace $\mathfrak{M}_2^{p,q}$. In (3), moreover, the subrepresentations $T_{p,q}$ with $p \leq -1$, $q \leq -1$ in $\mathfrak{M}_{p,q}^1 \cap \mathfrak{M}_{p,q}^2$, the subrepresentations $T_{p,-p}$ for $p \leq -1$ in $\mathfrak{M}_{p,-p}^2$, and $T_{p,-p}$ for $p \geq 1$ in $\mathfrak{M}_{|p|,-p}^1$ are indicated.

In (5) there are constructed (in infinitesimal form) the subrepresentations $T_{p,q}$ for $p \geq 1$, $q \leq -1$, $p + q \leq -1$ in $\mathfrak{M}_1^{p,q}$, the subrepresentations $T_{p,q}$ for $p \leq -1$, $q \geq 1$, $p + q \leq -1$ in $\mathfrak{M}_2^{p,q}$, and the subrepresentations $T_{p,q}$ for $p \leq -1$, $q \leq -1$ in $\mathfrak{M}_{p,q}^1 \cap \mathfrak{M}_{p,q}^2$. From the continuous series of unitary representations of the Lie algebra, given by the complex number σ (see item 6 of the addendum in (5)), only the representations with integral $\sigma + \bar{\sigma} = m$ extend to representations of the group. These are the representations $T^{\text{Im } \sigma, m}$ (reducible for $\text{Im } \sigma = 0$ and even m).

In conclusion I express my deep gratitude to Prof. M. A. Naimark for posing the problem and for advice.

Moscow State University
named after M. V. Lomonosov

Received
26 IV 1967

CITED LITERATURE

1. M. A. Naimark, *Linear Representations of the Lorentz Group*, Moscow, 1958.
2. M. I. Graev, DAN, 98, No. 4, 517 (1954).
3. M. I. Graev, DAN, 113, No. 5, 966 (1957).
4. M. I. Graev, Trudy Mosk. Mat. Obshch., 7, 335 (1958).
5. I. M. Gelfand, M. I. Graev, Izv. AN SSSR, Ser. Mat., 29, No. 6, 1329 (1965).
6. Harish-Chandra, Trans. Am. Math. Soc., 76, 1, 26 (1954).
7. S. Helgason, *Differential Geometry and Symmetric Spaces*, Moscow, 1964.

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.