

ON PERMISSIBILITY CONDITIONS FOR DISTRIBUTIONS OF THE MASSES OF UNSTABLE PARTICLES

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Abstract

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PHYSICS

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ON PERMISSIBILITY CONDITIONS FOR DISTRIBUTIONS OF THE MASSES OF UNSTABLE PARTICLES

(Presented by Academician V. A. Fock, 20 XI 1967)

1. Consider the reaction

$$m_1 + m_2 \rightarrow m + M \rightarrow m_4 + m_5 + m_6 + m_7, \quad (1)$$

where m_i are stable particles, while m and M are unstable particles (resonances). If, for reaction (1), the law of conservation of energy-momentum “from stable to stable” ^(1,2) is fulfilled, then the masses m and M turn out to be related to one another* ⁽³⁾

$$M^2 = m^2 + (E_0^2 - p_0^2) - 2E_0\sqrt{m^2 + p_m^2} + 2p_0p_m, \quad (2)$$

where $E_0 \equiv \sqrt{m_1^2 + p_1^2} + \sqrt{m_2^2 + p_2^2}$, $p_0 \equiv p_1 + p_2$ are the energy and momentum of the colliding particles, and $p_m \equiv p_4 + p_5$ is the momentum of particle m .

It follows from (2) that

$$\omega(m; p_m) dm = W(M; p_M) dM, \quad (3a)$$

$$\bar{\omega}(p_m) dp_m = \bar{W}(p_M) dp_M, \quad (3b)$$

where $\omega(m; p_m)$, $W(M; p_M)$ are the densities of the conditional mass distributions of the particles m and M , i.e., the densities, respectively, at fixed values of the momenta p_m and p_M , and $\bar{\omega}(p_m)$, $\bar{W}(p_M)$ are the densities of the distributions of these momenta. Since it is precisely these conditional mass distributions and momentum distributions that essentially determine the decay law of unstable particles ⁽³⁾, it follows from (3) that there exists a dynamical “filter” of masses and that one unstable particle influences the decay law of the other ⁽³⁾.

The unconditional mass distributions $\tilde{\omega}(m)$, $\tilde{W}(M)$, which enter into the S -matrix description of reaction (1), and, consequently, also of the direct reaction

$$m_1 + m_2 \rightarrow m_4 + m_5 + m_6 + m_7, \quad (1')$$

are related to the conditional distributions according to

$$\tilde{\omega}(m) = \int \bar{\omega}(p_m) \omega(m; p_m) dp_m, \quad (4a)$$

$$\tilde{W}(M) = \int \bar{W}(p_M) W(M; p_M) dp_M. \quad (4b)$$

It is precisely these unconditional mass distributions that are measured in experiments on the investigation of resonances.

2. In connection with (4), a natural question arises: are arbitrary, mutually independent mass distributions $\omega(m)$, $\tilde{W}(M)$ of associatively produced unstable particles permissible? Mathematically the problem reduces to the conditions for the existence, for arbitrary and mutually independent nonnegative normalized solutions $\omega(m)$ and $\tilde{W}(M)$, of the system (4), namely $\omega(p_m)$ ($\bar{W}(p_M)$), $\omega(m; p_m)$ ($W(M; p_M)$).

* To simplify the formulas, we assume, without loss of generality, that $(p_0, p_m) = p_0 p_m$.

From the point of view of the usual approach, with the separation of the interactions into strong and decay interactions, a positive answer seems predetermined and obvious.

If one of the particles, for example m , is stable,

$$\tilde{\omega}(m) = \omega(m; p_m) = \delta(m - m_0), \quad (5)$$

then, on the basis of (2),

$$\tilde{W}(M) dM = \omega(p_m) dp_m = \bar{W}(p_M) dp_M, \quad (6)$$

and, consequently, in this case arbitrary mass distributions $\tilde{W}(M)$ of singly produced unstable particles are indeed permissible (for example, $\gamma + p \rightarrow p + \pi^0$).

A completely different situation arises when both particles m and M are unstable. We shall seek a solution of the system (4), assuming

Fig. 1

Figure 1: Fig. 1

$$\tilde{\omega}(m) = \omega(m; p_m) = \frac{1}{\pi} \frac{\Gamma_m}{(m - m_0)^2 + \Gamma_m^2}. \quad (7)$$

Then (4a) is automatically satisfied, while (4b) becomes

$$\tilde{W}(M) = \int_0^{p_0} \tilde{\omega}(p_m) \tilde{\omega}(m(M, p_m)) \left| \frac{dm}{dM} \right| dp_m = \int_0^{p_0} \bar{\omega}(p_m) K(M, p_m) dp_m, \quad (8)$$

i.e., an integral equation of the first kind with respect to $\omega(p_m)$. The question of the admissibility of $\tilde{W}(M)$ is thus reduced to the question of the existence of a nonnegative normalized solution ($\omega(p_m) \geq 0$, $\int \omega(p_m) dp_m = 1 < \infty$) of the integral equation (8).

Fig. 1. $M_{0\min} = E_0 - p_0$; $M_{0\max} = \sqrt{E_0^2 - p_0^2}$; $\check{M}_{0\min} = E_0 - \sqrt{m_0^2 + p_0^2}$; $\check{M}_{0\max} = \sqrt{(E_0 - m_0)^2 - p_0^2}$; $\check{\Gamma}_{\min} = \check{\Gamma}_m m_0 / \sqrt{m_0^2 + p_0^2}$; $\check{\Gamma}_{\max} = \check{\Gamma}_m (E_0 - m_0) / \sqrt{(E_0 - m_0)^2 - p_0^2}$.

To answer this rather complicated question, let us study the behavior of the right-hand side of (8) in the plane of the complex variable M . More precisely, we shall examine the singularities of the kernel of equation (8):

$$K(M, p_m) = \frac{M}{m} \frac{\sqrt{m^2 + p_m^2}}{E_0 - \sqrt{m^2 + p_m^2}} \frac{1}{\pi} \frac{\Gamma_m}{(m - m_0)^2 + \Gamma_m^2}. \quad (9)$$

Assuming that $M = M_0 \pm i\Gamma_M$; $m = m_0 \pm i\Gamma_m$, and $\Gamma_m/m_0 \ll 1$, $\Gamma_M/M_0 \ll 1$, on the basis of (2) it is not difficult to obtain the relation between the singularities of the kernel K in the variable m and the singularities induced by them in the variable M :

$$M_0^2 = m_0^2 + (E_0^2 - p_0^2) - 2E_0 \sqrt{m_0^2 + p_m^2} + 2p_0 p_m, \quad (10a)$$

$$\Gamma_M = \pm \Gamma_m \frac{m_0}{M_0} \frac{E_0 - \sqrt{m_0^2 + p_m^2}}{\sqrt{m_0^2 + p_m^2}}. \quad (10b)$$

It is important to emphasize that, according to (10), the real (M_0) and imaginary (Γ_M) parts of the singularities induced in the complex variable M are related to one another.

Since $p_m \leq p_0 < \infty$, it follows that

$$M_0 < \infty; \quad \sqrt{m_0^2 + p_m^2} < \infty; \quad \sqrt{m^2 + p_m^2} < \infty \quad (11)$$

and, if $m_6 \neq 0$ or $m_7 \neq 0$, then

$$E_0 - \sqrt{m_0^2 + p_m^2} \neq 0; \quad E_0 - \sqrt{m^2 + p_m^2} \neq 0. \quad (12)$$

It follows from (11), (12), on the basis of (10), that the induced singularities in M do not intersect the real axis if the singularities in m do not intersect the real axis.

On the basis of (9), (10) one can show that the singularities of the kernel $K(M, p_m)$ in the complex M -plane are concentrated only on a finite segment of the real axis and on a finite curve C , which are shown in Fig. 1. Let us note that one can choose p_0, m_1, m_2, m_6, m_7 so that $M_{\min} = m_6 + m_7 < M_{0\min}$.

Since by definition $\tilde{\omega}(p_m) \in L_1$, on the basis of the singularities of the kernel $K(M, p_m)$ studied above one can assert:

Theorem 1. *A necessary condition for the existence of a normalized solution of equation (8), i.e., a necessary condition for the admissibility of an arbitrary mass distribution $\tilde{W}(M)$ for a fixed mass distribution $\tilde{\omega}(m) = \omega(m; p_m)$, is that the singularities of $\tilde{W}(M)$ in the M -plane be located on the curves C .*

From this theorem it follows:

Theorem 2. *If the mass distribution of the particle m is fixed (7), then the full mass distribution of the particle M*

$$\tilde{W}(M) = \frac{1}{\pi} \left(\frac{\Gamma_{M_1}}{(M - M_1)^2 + \Gamma_{M_1}^2} \right) \quad (13)$$

with arbitrary M_1 and Γ_{M_1} is inadmissible.

One can show that in fact a much more general

Theorem 3. *If $\omega(m; p_m)$, as a function of the complex variable m , admits continuation from the real axis and its singularities do not fill the whole plane of the complex variable m when p_m is varied, then a mass distribution $\tilde{W}(M)$ with arbitrary singularities in the plane of the complex variable M , independent of the singularities of $\omega(m, p_m)$ ($\tilde{\omega}(m)$), is inadmissible.*

The results obtained paradoxically contradict the usually assumed fair statements that the mass distributions of unstable elementary particles do not depend on their preparation (i.e., in particular, on what they are associatively produced with) and are determined only by the decay interaction.

In connection with the existence of a reaction of the type

$$\gamma + p \rightarrow n + \pi^+ \quad (14)$$

with $T_n \simeq 10^3$ sec., $T_{\pi^+} \simeq 10^{-8}$ sec., only the following alternative is possible:
a) in the reaction of formation of ordinary unstable particles (not resonances) the energy-momentum conservation law “from stable particles to stable particles” is violated ⁽²⁾, b) the mass distributions of unstable particles do not admit continuation into the complex plane and, in particular, the usually assumed pole mass distributions of unstable particles are inadmissible. Both alternatives are quite unexpected.

A detailed exposition, an investigation of sufficient conditions for the admissibility of mass distributions, a discussion of the resulting alternatives, and, in connection with alternative b), the problem of correctness for equation (8) ⁽⁴⁾, will be published separately.

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CITED LITERATURE

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² L. A. Khalfin, DAN, **165**, 541 (1965).

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⁴ V. N. Sudakov, L. A. Khalfin, DAN, **157**, 1058 (1964).

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