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Abstract

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PHYSICS

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RAYLEIGH WAVES IN CADMIUM SULFIDE CRYSTALS

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The propagation of longitudinal and transverse waves in semiconductor CdS crystals has been studied quite thoroughly ⁽¹⁾. The propagation of Rayleigh waves in such crystals (taking piezoelectric properties and conductivity into account) has not been considered theoretically. Meanwhile, the investigation of this question is of great interest for determining the characteristics of the surface layer of the crystal.

In the present work an attempt is made to solve the problem of the propagation of plane harmonic Rayleigh waves (with time dependence according to the factor $e^{-i\omega t}$) in a CdS crystal bordering on vacuum. The case of greatest practical interest is considered, when the Rayleigh wave propagates in the x direction in a plane perpendicular to the hexagonal z axis of the crystal (the z axis is directed into the crystal). In this case the characteristics of the Rayleigh wave do not depend on the direction of the x axis in this plane. Owing to the presence in a CdS crystal of piezoelectric properties and conductivity, the propagation of a Rayleigh wave in it is accompanied by an alternating electric field, through which the wave interacts with the conduction electrons of the crystal. The displacement components u_x and u_z in the wave must satisfy the following equations of motion and the piezoelectric equations

$$\rho \partial^2 u_i / \partial t^2 = \partial T_{ik} / \partial x_k; \quad (1)$$

$$T_{ik} = c_{iklm}^E u_{lm} - e_{jik} E_j, \quad D_n = 4\pi e_{nlm} u_{lm} + \varepsilon_{jn}^S E_j. \quad (2)$$

Here ρ is the density of the crystal; T_{ik} , u_{lm} , c_{iklm}^E are components of the tensors, respectively, of stress, strain, and elastic moduli (the latter taken at constant electric field); E_j and D_n are components of the electric-field and induction vectors; ε_{jn}^S are components of the dielectric-permittivity tensor at constant entropy; e_{jik} are components of the piezoelectric-constant tensor. In addition, Poisson's equation, the equation for the current density j , and the continuity equation must be satisfied. These equations may be written in the form

$$\partial D_n / \partial x_n = -4\pi en; \quad (3)$$

$$j_k = \sigma_{ik} E_i + e f D_{ik} \partial n / \partial x_i; \quad (4)$$

$$\partial j_k / \partial x_k - e \partial n / \partial t = 0, \quad (5)$$

where σ_{ik} and D_{ik} are components of the tensors of conductivity and of the electron diffusion coefficient; e is the electron charge; n is the excess of the electron concentration over its equilibrium value n_0 (in the absence of the wave); f is the trapping factor. Using the condition that the speed of sound is small in comparison with the speed of light, from the first two Maxwell equations one can easily show that, for the case considered by us, the electric field in the crystal may, to a very good degree of accuracy, be regarded as potential, i.e. $E_j = -\partial\psi/\partial x_j$, where ψ is the electric potential.

Combining equations (1)–(5), we obtain 3 equations for the unknowns u_x, u_z , and ψ :

$$\rho \partial^2 u_i / \partial t^2 = c_{iklm}^E \partial^2 u_m / \partial x_k \partial x_l + e_{jik} \partial^2 \psi / \partial x_k \partial x_j,$$

$$\frac{1}{2} e_{nlm} \frac{\partial^2}{\partial t \partial x_n} \left(\frac{\partial u_l}{\partial x_m} + \frac{\partial u_m}{\partial x_l} \right) - \frac{1}{2} f D e_{nlm} \frac{\partial^3}{\partial x_k^2 \partial x_n} \left(\frac{\partial u_l}{\partial x_m} + \frac{\partial u_m}{\partial x_l} \right) - \sigma_0 \frac{\partial^2 \psi}{\partial x_k^2} - \frac{\varepsilon_{jn}^S}{4\pi} \frac{\partial^3 \psi}{\partial t \partial x_j \partial x_n} + \frac{f D \varepsilon_{jn}^S}{4\pi} \frac{\partial^4 \psi}{\partial x_k^2 \partial x_j \partial x_n} = 0$$

The last equation has been linearized: in it we have neglected a term $\sim E_j^2$, which at ordinary energy-flux densities in a Rayleigh wave (less than 1 W/cm²) is quite justified. In addition, we have made one more generally accepted simplification, taking the electron diffusion coefficient and the conductivity of the crystal to be scalars: $D_{ik} = \delta_{ik} D$, $\sigma_{ik} = \delta_{ik} \sigma = \delta_{ik} \mu e (n_0 + f n)$, where μ is the mobility of electrons in the crystal, and $e n_0 \mu = \sigma_0$.

We shall seek the solution of equations (6) in the form

$$u_x = \frac{A}{k} e^{\beta k z + i(kx - \omega t)}, \quad u_z = \frac{B}{k} e^{\beta k z + i(kx - \omega t)}, \quad \psi = \frac{e_{113} C}{k} e^{\beta k z + i(kx - \omega t)}. \quad (7)$$

Here k is the as yet unknown wave number of the Rayleigh wave; β is a function of k ; A, B, C are dimensionless arbitrary constants. Substituting expressions (7) into equations (6), we obtain a system of 3 linear homogeneous algebraic equations for A, B, C . Equating the determinant of this system to zero, we shall have an equation for finding the functions $\beta(k)$. It can be shown that,

from the entire set of roots of this equation, 3 roots correspond to the Rayleigh wave; for convenience in the subsequent exposition it is expedient to represent them in the form

$$\beta_1 = \beta_1^0 + \delta_1, \quad \beta_2 = \beta_2^0 + \delta_2, \quad \beta_3 = 1 + \delta_3. \quad (8)$$

Here $\beta_{1,2}^0$ are the corresponding roots for the Rayleigh wave in a CdS crystal in the absence of the piezoelectric effect ($e_{ijk} = 0$). The values of all three roots were found by us by an approximate solution of the indicated equation. After calculating $\beta_{1,2,3}$, two of the three arbitrary constants A, B, C can be expressed through the third. As a result we obtain:

$$u_x = \frac{1}{k} \sum_{n=1}^3 A_n e^{\beta_n k z + i(kx - \omega t)}, \quad u_z = \frac{1}{k} \sum_{n=1}^3 F(\beta_n) A_n e^{\beta_n k z + i(kx - \omega t)},$$

$$\psi = \frac{e_{113}}{k} \sum_{n=1}^3 G(\beta_n) A_n e^{\beta_n k z + i(kx - \omega t)}. \quad (9)$$

Here $F(\beta_n), G(\beta_n)$ are functions of β_n and of the constants of the CdS crystal; A_1, A_2, A_3 are new arbitrary constants.

At the boundary of the crystal with vacuum (the plane $z = 0$), the boundary conditions of absence of stresses, continuity of the normal component of the electric induction vector, and of the tangential component of the electric-field vector must be satisfied. The last two conditions, strictly speaking, are applicable only to dielectrics, but for comparatively small conductivities of the crystal ($\sigma_0 < 10^{-1} \Omega^{-1} \cdot \text{cm}^{-1}$), which are usually realized experimentally, these conditions are also very well suited. For the electric potential ψ_0 in vacuum, from Poisson's equation $\Delta\psi_0 = 0$ and the boundary condition of continuity of the tangential component $\mathbf{E}$, we obtain the expression $\psi_0 = \psi(0)e^{-kz}$, where $\psi(0)$ is the value of the electric potential in the crystal at $z = 0$. The remaining three boundary conditions lead to the equations:

$$c_{1133}^E \partial u_x / \partial x + c_{3333}^E \partial u_z / \partial z + e_{333} \partial \psi / \partial z = 0,$$

$$c_{1313}^E (\partial u_x / \partial z + \partial u_z / \partial x) + e_{113} \partial \psi / \partial x = 0, \quad (10)$$

$$e_{333} \frac{\partial u_z}{\partial z} + e_{311} \frac{\partial u_x}{\partial x} - \frac{\varepsilon_{33}^S}{4\pi} \frac{\partial \psi}{\partial z} - \frac{k}{4\pi} \psi = 0,$$

where the values of all the functions and of their derivatives are taken at $z = 0$.

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

Substituting expressions (9) into equations (10) and setting equal to zero the determinant of the system of linear homogeneous equations obtained in this way, we shall have the dispersion equation for finding the wave number k of the Rayleigh wave.

This equation is very cumbersome. We shall seek its solution by the method of successive approximations, putting $k = k_0(1 + \alpha)$, where k_0 is the wave number of the Rayleigh wave in the crystal in the absence of the piezoelectric effect. Using for β_i expressions (8) and expanding the terms of the dispersion equation in powers of the small parameter e_{113}^2/c_{1313} , we obtain, in the zeroth approximation, an equation for k_0 . The next approximation gives an equation for α , from which, after a number of calculations, we obtain the following calculation formulas:

Fig. 1

$$\begin{aligned} \frac{\Delta c}{c_0} = -\operatorname{Re} \alpha = & \frac{0.232 \cdot 10^{-2} (\omega^2/\omega_D^2 + 1.04) (\omega^2/\omega_D^2 - 1.277\omega_c/\omega_D + 1.04)}{1.69\omega_c^2/\omega^2 + (\omega^2/\omega_D^2 - 1.277\omega_c/\omega_D + 1.04)^2} \\ & + \frac{1.64 \cdot 10^{-2} (\omega^2/\omega_D^2 + 1.065) (\omega^2/\omega_D^2 + 1.438\omega_c/\omega_D + 1.065)}{2.21\omega_c^2/\omega^2 + (\omega^2/\omega_D^2 + 1.438\omega_c/\omega_D + 1.065)^2}, \quad (11) \\ \gamma = \operatorname{Im} \alpha = & \frac{0.302 \cdot 10^{-2} (\omega_c/\omega) (\omega^2/\omega_D^2 + 1.04)}{1.69\omega_c^2/\omega^2 + (\omega^2/\omega_D^2 - 1.277\omega_c/\omega_D + 1.04)^2} \\ & + \frac{2.435 \cdot 10^{-2} (\omega_c/\omega) (\omega^2/\omega_D^2 + 1.065)}{2.21\omega_c^2/\omega^2 + (\omega^2/\omega_D^2 + 1.438\omega_c/\omega_D + 1.065)^2}. \end{aligned}$$

Here the notation is as follows: $\Delta c/c_0$ is the relative change in the phase velocity of the Rayleigh wave due to the piezoelectric effect and the conductivity of the crystal; γ is the attenuation coefficient of the Rayleigh wave per wavelength, referred to 2π ; $\omega_c = \sigma_0$; $\omega_D = c_0^2/fD$. In deriving these formulas we used the values of the constants of the CdS crystal taken from [2]. The electron mobility μ was taken equal to $200 \text{ V}^{-1} \cdot \text{cm}^2 \cdot \text{s}^{-1}$, $T = 300^\circ\text{K}$; the trap factor f was taken equal to 1. The maximum error of formulas (11) does not exceed 25%.

Fig. 2

Fig. 3

Figure 3: Fig. 3

Figures 1 and 2 show the dependences, calculated from formulas (11), of $\Delta c/c_0$ (curves 1) and γ (curves 2) on frequency on a semilogarithmic scale. The curves were calculated for two limiting cases: $\omega_D = \infty$ (Fig. 1) and $\omega_D = \omega_c$ (Fig. 2). In the experiments described in the literature, as a rule, $\omega_D > \omega_c$. As is seen from the figures, to each value of the crystal conductivity ω_c there corresponds a frequency at which the interaction of the Rayleigh wave with the electrons is maximal: the attenuation γ and the change in velocity are maximal.

As the electron diffusion increases (as ω_D decreases), the interaction weakens. The indicated dependences qualitatively coincide with analogous dependences for longitudinal and transverse waves in CdS ⁽¹⁾.

Figure 3 gives calculated curves of the dependences of $\Delta c/c_0$ (curve 1) and of the attenuation coefficient γ over a path of 1 cm (curve 2) on the logarithm of the conductivity of the crystal at a frequency of 30 MHz.

Fig. 3

The dots and circles denote the experimental values of γ at this frequency, taken from ^(3,4), under illumination of the crystal by a DRSh-500 mercury lamp and a K-22 projection lamp, respectively. Comparing the curve $\gamma(\lg \sigma)$ with analogous curves for longitudinal and transverse waves ⁽¹⁾, one can readily see that the ranges of conductivity values corresponding to strong interaction of the wave with electrons (large γ) coincide for all types of waves. As is further seen from Fig. 3, the principal difference between the experimental and calculated attenuation curves is their displacement relative to one another. In ⁽⁴⁾ it was suggested that the displacement of the attenuation curves of Rayleigh waves, depending on the spectral composition of the illumination and in comparison with the attenuation curves for longitudinal and transverse waves, is explained by the difference between the state of the surface layer of the crystal and the state of the bulk. The fact that theoretically the regions of strong interaction of the wave with electrons are the same for all types of waves confirms this suggestion.

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