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Abstract

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PHYSICS

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ABSORPTION OF ULTRASOUND BY FREE ELECTRONS OF A METAL IN A QUANTIZING MAGNETIC FIELD

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In connection with the difficulties of integrating the Boltzmann equation in the problem of the absorption of ultrasound by the electrons of a metal, the collision integral is usually approximated by the τ -approximation. This means that the collision integral is replaced by the expression

$$(f_0 - f)/\tau, \quad (1)$$

where τ is the relaxation time; f_0 is the locally equilibrium electron distribution function (for example, the Fermi function) and has the form

$$f_0 \left(\left[\frac{m}{2} (\mathbf{v} - \dot{\mathbf{U}})^2 - \xi(\mathbf{r}, t) \right] / T(\mathbf{r}, t) \right). \quad (2)$$

Here m is the electron mass; $\dot{\mathbf{U}}(\mathbf{r}, t)$, $\xi(\mathbf{r}, t)$, and $T(\mathbf{r}, t)$ are the local values of the lattice velocity, the chemical potential of the electrons, and their temperature, respectively. Such an approximation of the collision integral leads to the following formula for the dissipation of ultrasound energy ⁽¹⁾

$$Q_{cl} = -\frac{1}{2} \operatorname{Re} \left(\mathbf{E} \mathbf{j}_p^* - \frac{N_{01} m}{\tau} (\mathbf{V} - \dot{\mathbf{U}}) \dot{\mathbf{U}}^* \right); \quad (3)$$

here $\mathbf{j}_p^* = e_2 N_{02} \mathbf{U}^*$; N_{01} and N_{02} are the mean density of electrons and ions; e_2 is the ion charge; $\mathbf{V}$ is the local macroscopic velocity of the electrons; $\mathbf{E}$ is the intensity of the self-consistent electric field.

At low temperatures in a strong magnetic field H , when $\hbar\Omega \gg T$ and $\Omega\tau \gg 1$ (Ω is the cyclotron frequency of the electrons, and τ is their relaxation time), quantization of the orbital motion of the electrons becomes significant. Under these conditions the Boltzmann kinetic equation is inapplicable, and the state of the electrons should be described by the density matrix $\hat{\rho}$. Naturally, in the

τ -approximation, in the right-hand side of the equation of motion for $\hat{\rho}$ one should write an expression analogous to (1); then

$$\frac{\partial}{\partial t} \hat{\rho} + \frac{i}{\hbar} [\hat{H}, \hat{\rho}] = (\hat{\rho}_0 - \hat{\rho})/\tau, \quad \hat{H} = \left(\mathbf{P} - \frac{e_1}{c} \mathbf{A} \right)^2 / 2m + e_1 \varphi. \quad (4)$$

Just as in the classical theory, here $\hat{\rho}$ must relax to the locally equilibrium operator $\hat{\rho}_0$, defined by the formula

$$\hat{\rho}_0 = \rho_0 \left(\left\{ \left[\left(\hat{\mathbf{P}} - \frac{e_1}{c} \mathbf{A} - m \dot{\mathbf{U}} \right)^2 / 2m + e_1 \varphi - \xi(\mathbf{r}, t) \right], T^{-1}(\mathbf{r}, t) \right\}_+ \right), \quad (5)$$

where $\mathbf{A} = \mathbf{A}_0 + \mathbf{A}'$; $\mathbf{A}_0$ is the vector potential of the constant magnetic field; $\mathbf{A}'$ and φ are the electromagnetic potentials of the self-consistent field; $\{\hat{A}, \hat{B}\}_+ = (\hat{A}\hat{B} + \hat{B}\hat{A})/2$. We note that the inclusion of the scalar potential φ in $\hat{\rho}_0$ leads to a change of the chemical potential $\xi(\mathbf{r}, t)$ in such a way that the quantity $(e\varphi - \xi(\mathbf{r}, t))$ entering into (5) does not depend on φ (2). It is precisely for this reason that equation (4) proves to be invariant under a gauge transformation.

transformations of the electromagnetic potentials. In what follows it is convenient to use the calibration of the potentials $\varphi = 0$, $\mathbf{H} = \text{rot } \mathbf{A}$, $\mathbf{E} = -\frac{1}{c} \dot{\mathbf{A}}$; in this case the field equations have the form

$$\text{rot rot } \mathbf{E} = -\frac{4\pi}{c^2} \frac{\partial}{\partial t} \left(\frac{1}{4\pi} \dot{\mathbf{E}} + \mathbf{j} \right), \quad \text{div } \mathbf{E} = 4\pi(e_1 N_1 + e_2 N_2) \quad (6)$$

($e_1 N_1$ and $e_2 N_2$ are the charge densities of the electrons and ions, respectively). If the dependence of the temperature on coordinates and time is neglected, then, with the chosen calibration, instead of (5) we obtain

$$\hat{\rho}_0 = \rho_0 \left(\left[\left(\hat{\mathbf{P}} - \frac{e_1}{c} \mathbf{A} - m \dot{\mathbf{U}} \right)^2 / 2m - \xi(\mathbf{r}, t) \right] / T \right), \quad (7)$$

where we shall take $\xi(\mathbf{r}, t) = \xi + \delta\xi(\mathbf{r}, t)$.

In calculating the dissipation of ultrasonic energy it is necessary to know the forces acting on the lattice when its interaction with the electrons is taken into account. There will be two such forces: the Lorentz force $\mathbf{F}_L = e_2 N_{02} (\mathbf{E} + \frac{1}{c} [\mathbf{U} \times \mathbf{H}])$ and the force $\mathbf{F}_{st}$, caused by collisions of the electrons with the lattice. To calculate $\mathbf{F}_{st}$, one must multiply $(\rho_0 - \rho)/\tau$ by the electron momentum-density operator $m\{\hat{\mathbf{v}}, \hat{N}_1\}$ ($\hat{\mathbf{v}}$ is the electron velocity operator, and $\hat{N}_1$ is the density operator) and take the trace. In the linear approximation in $\dot{\mathbf{U}}$, $\mathbf{A}'$, $\delta\xi$, we find

$$\mathbf{F}_{st} = \frac{m}{\tau e_1} (\mathbf{j}^{(e)} - N_{01} e_1 \dot{\mathbf{U}}) - \frac{m}{\tau e_1} \mathbf{j}^{(0e)}. \quad (8)$$

The Fourier components $\mathbf{j}^{(e)}$ and $\mathbf{j}^{(0e)}$ are expressed by the formulas

$$\mathbf{j}^{(e)} = e_1 \sum_{\nu\nu'} \rho_{\nu\nu'} \mathbf{V}_{\nu'\nu} \equiv e_1 N_{01} \mathbf{V}, \quad (9)$$

$$j_\alpha^{(0e)} = - \left(\frac{\sigma_0}{c\tau} \delta_\beta + \frac{e_1^2}{c} \sum_{\nu,\nu'} \frac{\rho_{0\nu'} - \rho_{0\nu}}{E_{\nu'} - E_\nu} \dot{V}_{\nu\nu'}^{(\alpha)} V_{\nu'\nu}^{(\beta)} \right) \left(A'_\beta + \frac{mc}{e_1} \dot{U}_\beta \right), \quad (10)$$

$$V_{\nu\nu'}^{(\alpha)} = \frac{1}{2} \langle \nu | e^{i\mathbf{q}\mathbf{r}} v_\alpha + \hat{v}_\alpha e^{i\mathbf{q}\mathbf{r}} | \nu' \rangle, \quad \sigma_0 = \frac{e_1^2 N_{01}}{m} \tau; \quad (11)$$

$|\nu\rangle$ and E_ν are determined by the equations

$$\left(p - \frac{e_1}{c} A_0 \right)^2 / 2m |\nu\rangle = E_\nu |\nu\rangle, \quad (12)$$

$$\rho_{0\nu} = \left[\exp \left(\frac{E_\nu - \xi}{T} \right) + 1 \right]^{-1}. \quad (13)$$

The equation of motion of the lattice will have the form

$$MN_{02} \ddot{U}_i - \lambda_{iklj}^0 \frac{\partial^2 u_j}{\partial x_k \partial x_l} = (F_{st})_i + (F_L)_i \quad (14)$$

(λ_{iklj}^0 is the tensor of the elastic moduli of the lattice without taking into account its interaction with the electrons; MN_{02} is the density of matter).

From (14) there follows the formula for the dissipation of ultrasonic energy

$$Q = -\frac{1}{2} \operatorname{Re} \left(\mathbf{E} \mathbf{j}_p^* - \frac{N_{01} m}{\tau} (\mathbf{V} - \dot{\mathbf{U}}) \dot{\mathbf{U}}^* + \frac{m}{e_1 \tau} \mathbf{j}^{(0e)} \dot{\mathbf{U}}^* \right). \quad (15)$$

In works (3), in order to calculate the coefficient of absorption of ultrasound in a quantizing magnetic field, formula (3) was used with the replacement of classical quantities by quantum ones. With such a generalization, (3) could not the last term in (15), which vanishes in the classical limit but plays an essential role in the quantum case, must be taken into account. It is now not difficult to find the ultrasound absorption coefficient $\Gamma(\omega, q)$. First, by means of the normalization condition $N_1 = \operatorname{sp}(\rho_0, \hat{N}_1)$ and the continuity equation $e_1 \dot{N}_1 +$

$\text{div } j^{(e)} = 0$, we express $\delta\zeta(\mathbf{r}, t)$ in terms of $j^{(e)}$. Then we integrate equation (4), in the linear approximation in $\dot{\mathbf{U}}$, $\mathbf{A}'$, $\delta\zeta$. Using these results and (9), we find the expression for the electron charge-current density:

$$j_{\alpha}^{(e)} = \left\{ m'_{\alpha\beta} E_{\beta} + \sigma'_{\alpha\beta} \left(E_{\beta} + \frac{m}{e_1 \tau} \dot{U}_{\beta} \right) \right\} \sigma_0, \quad (16)$$

where

$$\begin{aligned} m'_{\alpha\beta} &= D_{\alpha\gamma} m_{\gamma\beta}, & \sigma'_{\alpha\beta} &= D_{\alpha\gamma} \sigma_{\gamma\beta}, & D_{\alpha\gamma}^{-1} D_{\gamma\beta} &= \delta_{\alpha\beta}, \\ D_{\alpha\beta}^{-1} &= \delta_{\alpha\beta} + \frac{\sigma_0}{i\omega} \frac{R_0}{e_1^2 (1 + i\tau\omega)} (m_{\alpha\gamma} + \sigma_{\alpha\gamma}) q_{\gamma} q_{\beta}, \\ \sigma_0 m_{\alpha\beta} &= \frac{\sigma_0}{i\omega\tau} \delta_{\alpha\beta} + \frac{e_1^2}{i\omega} \sum_{\nu\nu'} \frac{\rho_{0\nu'} - \rho_{0\nu}}{E_{\nu'} - E_{\nu}} V_{\nu'\nu}^{*(\alpha)} V_{\nu'\nu}^{(\beta)}, \\ \sigma_0 \sigma_{\alpha\beta} &= -\frac{e_1^2}{i\omega} \sum_{\nu, \nu'} \left(\frac{\rho_{0\nu'} - \rho_{0\nu}}{E_{\nu'} - E_{\nu}} \right) \frac{i\omega\tau}{1 + i\tau(\omega - \omega_{\nu'\nu})} V_{\nu'\nu}^{*(\alpha)} V_{\nu'\nu}^{(3)}, \\ R_0 &= \left(-\sum_{\nu\nu'} \frac{\rho_{0\nu'} - \rho_{0\nu}}{E_{\nu'} - E_{\nu}} I_{\nu\nu'} \right)^{-1}, \\ I_{\nu\nu'} &= |\langle \nu | e^{i\mathbf{q}\mathbf{r}} | \nu' \rangle|^2, & \omega_{\nu\nu'} &= (E_{\nu'} - E_{\nu})/\hbar. \end{aligned} \quad (17)$$

The electromagnetic-field equation (6) makes it possible to express $\mathbf{E}$ in terms of $\dot{\mathbf{U}}$:

$$E_{\alpha} = \frac{e_2 N_{02}}{\sigma_0} w_{\alpha\beta} \dot{U}_{\beta} = \frac{e_2 N_{02}}{\sigma_0} (B_{\alpha\gamma} + m'_{\alpha\gamma} + \sigma'_{\alpha\gamma})^{-1} (\sigma'_{\gamma\beta} - \delta_{\gamma\beta}) \dot{U}_{\beta}. \quad (18)$$

The tensor $B_{\alpha\beta}$, in a coordinate system with the X -axis directed along the wave vector $\mathbf{q}$, has the form

$$B_{\alpha\beta} = \begin{pmatrix} -i\gamma & 0 & 0 \\ 0 & i\beta & 0 \\ 0 & 0 & i\beta \end{pmatrix}, \quad \beta = \frac{\omega c^2}{4\pi\sigma_0 v_s^2}, \quad \gamma = \beta \left(\frac{v_s}{c} \right)^2, \quad (19)$$

where v_s is the sound velocity.

Next, Q can be expressed, using (16), (17), in terms of $\dot{U}_\alpha$. Dividing Q by $(v_s N_{02} M |\dot{\mathbf{U}}|^2/2)$, we find the sound absorption coefficient

$$\Gamma = \frac{e_2}{e_1} \frac{m}{M} \left(\frac{1}{v_s \tau} \right) \operatorname{Re} \left\{ \dot{U}_\alpha^* [(\delta_{\alpha\gamma} + B_{\alpha\gamma} + m_{\alpha\gamma}) w_{\gamma\lambda} + i\omega\tau m_{\alpha\beta}] \dot{U}_\beta \right\}, \quad (20)$$

where $\dot{U}_\alpha^* = \dot{U}_\alpha^*/|\dot{\mathbf{U}}|$.

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