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ON A CERTAIN CLASS OF PROBLEMS IN INTEGRAL GEOMETRY

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Abstract

Full Text

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MATHEMATICS

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ON A CERTAIN CLASS OF PROBLEMS IN INTEGRAL GEOMETRY

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Let $\Gamma(x_1, x_2)$ be a two-parameter family of curves in the half-plane $y > 0$, satisfying the following conditions:

- 1) Through any pair of points of the half-plane with coordinates (x_1, y_1) , (x_2, y_2) , $x_1 \neq x_2$, there passes one and only one curve of the family. Moreover, through the pair of points $(x_1, 0)$, $(x_2, 0)$ there passes the curve $\Gamma(x_1, x_2)$.
- 2) If $x'_1 < x_1 < x_2 < x'_2$, then the part of the half-plane bounded by the curve $\Gamma(x_1, x_2)$ lies strictly inside the part of the half-plane bounded by the curve $\Gamma(x'_1, x'_2)$.
- 3) The functions defining the equations of the curves Γ in normal form are three times continuously differentiable with respect to arc length and to the parameters x_1, x_2 .

Further, let $u(x, y)$ be a function continuous in the half-plane, and let

$$v(x_1, x_2) = \int_{\Gamma(x_1, x_2)} u(x, y) ds.$$

Consider the following problem of integral geometry: given the function $v(x_1, x_2)$, it is required to determine the function $u(x, y)$.

A number of inverse problems for partial differential equations reduce to the formulated problem of integral geometry (see ^(3,4)). In the work of V. G. Romanov ⁽⁴⁾, analogous problems of integral geometry were investigated by the method of moments.

In the present note we give a formulation and a scheme of proof of the uniqueness theorem for the posed problem, without those additional restrictions on the families of curves Γ that are contained in ⁽⁵⁾.

Theorem. The function $u(x, y)$ is uniquely determined in the part of the half-plane $y > 0$ bounded by the curve $\Gamma(x_1^0, x_2^0)$, $x_1^0 < x_2^0$, by the values of the function $v(x_1, x_2)$ in the square $x_1^0 \leq x_1 \leq x_2^0$, $x_1^0 \leq x_2 \leq x_2^0$.

We give the scheme of the proof of the theorem. Consider some point (x, y) lying on the curve $\Gamma(x_1, x_2)$, and denote by θ the angle formed by the tangent to the curve $\Gamma(x_1, x_2)$ at the point (x, y) with the x -axis, $x_1 < x_2$. Denote by $\tilde{\Gamma}(x, y, \theta)$ the part of the curve $\Gamma(x_1, x_2)$ lying between the points $(x_1, 0)$, (x, y) , and consider the function

$$w(x, y, \theta) = \int_{\tilde{\Gamma}(x, y, \theta)} u(\xi, \eta) ds.$$

It is not difficult to verify that the function w satisfies the following second-order differential equation:

$$\frac{\partial}{\partial \theta} \left(\cos \theta \frac{\partial}{\partial x} + \sin \theta \frac{\partial}{\partial y} - K \frac{\partial}{\partial \theta} \right) w = 0, \quad (1)$$

where $K(x, y, \theta)$ is the curvature of the curve $\Gamma(x_1, x_2)$ at the point (x, y) .

The differential operator in equation (1) can be brought to the form

$$\begin{aligned} & \left(K \frac{\partial}{\partial \theta} - \frac{1}{2} \cos \theta \frac{\partial}{\partial x} - \frac{1}{2} \sin \theta \frac{\partial}{\partial y} \right)^2 - \\ & - \frac{1}{4} (\cos \theta \frac{\partial}{\partial x} + \sin \theta \frac{\partial}{\partial y})^2 + \\ & + K (\sin \theta \frac{\partial}{\partial x} - \cos \theta \frac{\partial}{\partial y}) + (\cos \theta K_x + \sin \theta K_y) \frac{\partial}{\partial \theta}. \end{aligned} \quad (2)$$

Let us consider, instead of the variables x, y, θ , new variables t, α, β such that t, α, β , as functions of x, y, θ , satisfy the equations

$$K \frac{\partial t}{\partial \theta} - \frac{1}{2} \cos \theta \frac{\partial t}{\partial x} - \frac{1}{2} \sin \theta \frac{\partial t}{\partial y} = 0,$$

$$\cos \theta \frac{\partial \alpha}{\partial x} + \sin \theta \frac{\partial \alpha}{\partial y} = 0,$$

$$\cos \theta \frac{\partial \beta}{\partial x} + \sin \theta \frac{\partial \beta}{\partial y} = 0.$$

In the variables t, α, β , equation (1) takes the form:

$$a_0 \frac{\partial^2 w}{\partial t^2} - (a_1 \frac{\partial}{\partial \alpha} + a_2 \frac{\partial}{\partial \beta})^2 w + a_3 \frac{\partial w}{\partial t} +$$

$$+a_4 \partial w / \partial \alpha + a_5 \partial w / \partial \beta = 0, \quad (3)$$

where the a_j are certain functions.

With a suitable choice of the variables t, α, β , the integral-geometry problem under consideration is reduced to the Cauchy problem for equation (3). The Cauchy data are prescribed on the surface

$$t = \varphi(\alpha, \beta),$$

where φ is a function with positive second differential.

The formulated uniqueness theorem follows, after the transformations indicated above, from Hörmander's general theorem (see (2)).

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REFERENCES

- ¹ I. M. Gelfand, M. I. Graev, N. Ya. Vilenkin, *Integral Geometry and Related Questions of Representation Theory*, Moscow, 1967.
- ² L. Hörmander, *Linear Partial Differential Operators*, Moscow, 1965.
- ³ M. M. Lavrent'ev, V. G. Romanov, DAN, 171, No. 6 (1966).
- ⁴ V. G. Romanov, DAN, 173, No. 4 (1967).
- ⁵ V. G. Romanov, *Siberian Mathematical Journal*, 8 (1967) (in press).

Note: Figure translations are in progress. See original paper for figures.

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