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Abstract

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MOTION OF PARTICLES SUSPENDED IN A TURBULENT FLOW

In the present paper we shall consider the question of the motion of solid particles relative to the turbulent gas in which they are suspended. However, the results obtained can also be applied to the motion of solid particles, undeformed drops and bubbles in a liquid, as well as small drops in a flow of turbulent gas. The subsequent discussion will concern only the case in which the specific volume occupied by the particles γ is small in comparison with unity.

We shall assume, moreover, that the influence of the particles on the motion of the gas can be neglected. The limits of applicability of this assumption will be considered in detail in the following paper. The turbulent flow will be assumed locally homogeneous, isotropic, and stationary, and the size of the suspended particles R so much smaller than the inner scale of turbulence l that the Reynolds numbers of the relative motion of the particles and the gas are small in comparison with unity. The equation of motion of particles satisfying this condition was written down by Chen and then refined by Corrsin and Lumley⁽¹⁾. This equation relates the velocity of motion of a particle in the gas W_i to the velocity of motion of the gas U_i surrounding the particle. It is, generally speaking, a nonlinear partial differential equation. However, this equation becomes a linear integro-differential equation, in which the only independent variable is time, if the inequalities

$$\frac{R^2}{\nu} \frac{\partial U}{\partial x} \ll 1, \quad \frac{W}{\nu} \frac{\partial U}{\partial x} / \frac{\partial^2 U}{\partial x^2} \gg 1,$$

are satisfied, where ν is the kinematic viscosity of the gas.

Let U_λ and ω_λ be the velocities and frequencies of pulsations whose size is of order λ . Taking into account that the particle velocity is close to U_L , the velocity of the largest gas pulsations, which have scale L of the order of the dimensions of the system, let us estimate the terms entering into the inequalities:

$$\frac{W}{\nu} \frac{\partial U}{\partial x} / \frac{\partial^2 U}{\partial x^2} \sim \frac{W}{\nu} \frac{U_\lambda}{\lambda} / \frac{U_\lambda}{\lambda^2} \sim \frac{U_L \lambda}{\nu} > \frac{U_L L}{\nu} \frac{l}{L} \sim \text{Re}_L^{1/4} \gg 1,$$

$$\frac{R^2}{\nu} \frac{\partial U}{\partial x} \sim \frac{R^2}{\nu} \frac{U_\lambda}{\lambda} \sim \frac{R^2}{\nu} \omega_\lambda < \frac{R^2}{\nu} \frac{\nu}{l^2} \sim \frac{R^2}{l^2} < 1.$$

Thus, from the estimates given above it follows that, under the condition $R < l$, the equation of steady relative motion of the particles has the form

$$\frac{dV_i}{dt} = (\alpha - 1) \frac{dU_i}{dt} - \beta V_i - \sqrt{\frac{3\alpha\beta}{\pi}} \int_{-\infty}^t \frac{dV_i}{d\tau} \frac{d\tau}{\sqrt{t-\tau}} + g_i. \quad (1)$$

In formula (1) the following notation has been used: $V_i = W_i - U_i$ is the velocity of motion of the particle relative to the gas; $\alpha = 3\rho_0/(2\rho + \rho_0)$, where ρ_0 is the density of the gas and ρ the density of the particle; g_i is the acceleration of free fall; $\beta = k_1\alpha\nu/R^2$ is a characteristic frequency determining the motion of particles in the gas and is the reciprocal of the time required for the gas flowing around the particle to bring it out of the state of rest. The quantity k_1 , as well as the quantities $k_2, k_3, \dots$ encountered below, are numerical constants of order

of the order of unity. By virtue of the linearity of equation (1), the velocity V_i is a superposition of two independent quantities: the velocity due to the presence of the gravitational field, and the velocity associated with the motion of the gas. Since in the present work we shall be interested in the motion of a particle caused by the motion of the gas, the quantity g_i in equation (1) may be omitted. However, because this equation is linear, the results obtained will nevertheless remain valid in the presence of a gravitational field. Under our assumption $R < l$, equation (1) admits further simplifications. It can be shown⁽²⁾ that the integral term in equation (1) is small in comparison with the others if, for all frequencies of the turbulent spectrum, the inequality $\omega_\lambda R^2/\nu \ll 1$ holds. This inequality, as is not difficult to see, is equivalent to the inequality $R^2 < l^2$. Indeed:

$$\omega_\lambda R^2/\nu < \omega_l R^2/\nu \sim R^2/l^2.$$

Thus, the equation of the relative motion of a particle in a gas, under our assumptions, has the form

$$\frac{dV_i}{dt} = (\alpha - 1) \frac{dU_i}{dt} - \beta V_i. \quad (2)$$

In a turbulent flow the velocities U_i and V_i are random functions of time. To determine the characteristics of the motion of particles relative to the gas it

is necessary to know the relation between the Lagrangian temporal correlation of the quantity U_i and the Lagrangian temporal correlation of the quantity V_i . Therefore let us establish the relation between the correlation functions

$$Q = \overline{U_i(t)U_i(t+\tau)}, \quad S = \overline{V_i(t)V_i(t+\tau)}.$$

The bar denotes averaging over all states of the system. For the problem under consideration, by virtue of the stationarity condition, Q and S are functions only of the correlation time τ and do not depend on t .

To find the relation between $Q(\tau)$ and $S(\tau)$, we expand U_i and V_i in Fourier integrals and then, using equation (2), establish the relation between the spectral function $\Phi(\omega)$ of the velocity field V_i and the spectral function $\Psi(\omega)$ of the velocity field U_i :

$$\Phi(\omega) = (1-\alpha)^2 \frac{\omega^2}{\omega^2 + \beta^2} \Psi(\omega). \quad (3)$$

Since the correlation function is simply the Fourier transform of the corresponding spectral function, taking formula (3) into account one may write

$$S(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi(\omega) e^{i\omega\tau} d\omega = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{(1-\alpha)^2 \omega^2}{\omega^2 + \beta^2} \Psi(\omega) e^{i\omega\tau} d\omega. \quad (4)$$

In order to establish the relation between the integral in formula (4) and the correlation function $Q(\tau)$, it is necessary to use Parseval's formula. Since

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{\omega^2}{\omega^2 + \beta^2} e^{i\omega\tau} d\omega = \delta(\tau) - \frac{\beta}{2} e^{-\beta|\tau|}$$

and since

$$\int_{-\infty}^{\infty} \Psi(\omega) e^{i\omega\tau} d\omega = Q(\tau),$$

then

$$S(\tau) = (1-\alpha)^2 \left[Q(\tau) - \frac{\beta}{2} \int_{-\infty}^{\infty} Q(\zeta) e^{-\beta|\tau-\zeta|} d\zeta \right]. \quad (5)$$

Using the evenness of the function $Q(\xi)$, formula (5) can be represented in the form

$$S(\tau) = (1 - \alpha)^2 \left[Q(\tau) - \frac{\beta}{2} \int_0^\infty Q(\xi) (e^{-\beta(\tau+\xi)} + e^{-\beta(\tau-\xi)}) d\xi \right]. \quad (6)$$

It is seen from expression (6) that, in order to determine $S(\tau)$, one must know the Lagrangian time correlation of the gas velocities in the given flow. Since, according to the assumption made earlier, the presence of particles in the flow does not affect the motion of the gas, the correlation function $Q(\tau)$ of the gas containing no suspended particles should be substituted into formula (6). Generally speaking, there is no theoretical formula for $Q(\tau)$ applicable for all correlation times. However, one can use an empirical formula that approximately describes the behavior of the correlation function. For large Reynolds numbers of the gas motion, the exponential function should be chosen as such a formula^(3,4). Consequently, if it is assumed that $Q(\tau) = Q(0) \exp\{-\omega_L \tau\}$, then from formula (6) one can determine $S(\tau)$:

$$S(\tau) = (1 - \alpha)^2 Q(0) \frac{\omega_L^2}{\omega_L^2 - \beta^2} \left[e^{-\omega_L \tau} - \frac{\beta}{\omega_L} e^{-\beta \tau} \right]. \quad (7)$$

Knowing the correlation function of the relative motion of the particles, one can determine the characteristics of their motion relative to the gas. The most important such characteristics are the root-mean-square velocity $(\overline{V^2})^{1/2}$ and the mean amplitude of oscillations a . From the definition of the correlation functions it follows that $Q(0) = \overline{U^2}$, $S(0) = \overline{V^2}$, and therefore

$$\overline{V^2} = \overline{U^2} \frac{(1 - \alpha)^2 \omega_L}{\omega_L + \beta} \simeq \overline{U^2} \frac{(1 - \alpha)^2 \omega_L}{\omega_L + \beta}. \quad (8)$$

It follows directly from formula (8) that if the particles are suspended in a gas flow ($\alpha \sim 10^{-3}$), and their dimensions are not too small (so that $\beta \ll \omega_L$), then the root-mean-square velocity of the relative motion of the particles will coincide with the mean velocity of the turbulent pulsations of the gas. If, however, the particles are suspended in a turbulent liquid, then in this case always $\beta \gg \omega_L$, and the root-mean-square velocity of the relative motion $(\overline{V^2})^{1/2}$ is always much smaller than the root-mean-square velocity of the liquid $(\overline{U^2})^{1/2}$.

Since in a turbulent flow all frequencies of the turbulence spectrum affect the motion of a particle, it is obvious that by the mean amplitude of particle oscillations one should understand the root-mean-square displacement of the particle relative to the gas $[\overline{r^2(t)}]^{1/2}$ over a sufficiently long time, i.e., as $t \rightarrow \infty$. Since the quantities r_i and V_i are related by

$$r_i(t) = \int_0^t V_i(t') dt'$$

and the random function $V_i(t)$ is stationary, one can express the root-mean-square displacement $[\overline{r^2(t)}]^{1/2}$ through the correlation function $S(\tau)$ [5]:

$$\overline{r^2(t)} = 2 \int_0^t (t - \tau) S(\tau) d\tau. \quad (9)$$

Despite the stationarity of the function $V_i(t)$, the root-mean-square displacement of the particles, as is seen, is a function of time. This circumstance is due to the fact that the integral of a stationary random function does not, generally speaking, possess the property of stationarity ⁽⁵⁾. Substituting into (9) the function $S(\tau)$ defined by formula (7), and carrying out the integration,

we obtain

$$\overline{r^2(t)} = (1 - \alpha)^2 \frac{2\overline{U^2}}{\omega_L^2 - \beta^2} \left[\frac{\omega_L}{\beta} (1 - e^{-\beta t}) - (1 - e^{-\omega_L t}) \right], \quad (10)$$

$$a^2 = \lim_{t \rightarrow \infty} \overline{r^2(t)} = \frac{2\overline{U^2}}{\beta(\omega_L + \beta)} \simeq \frac{2U_L^2}{\beta(\omega_L + \beta)}. \quad (11)$$

Comparing the quantity a with the size of the pulsations having the largest scale $L \sim U_L/\omega_L$, we obtain

$$a/L \sim \omega_L / \sqrt{\beta(\omega_L + \beta)}. \quad (12)$$

Formula (12) shows that, in the case of particle motion in a gas flow under the condition $\overline{V^2} = \overline{U^2}$, the quantity a can considerably exceed L . For the opposite limiting case $\beta \gg \omega_L$, the root-mean-square displacement of the particles a will be much smaller than the dimensions of the largest turbulent pulsations L . In this case the motion of a particle suspended in a turbulent flow, over sufficiently long time intervals, may be regarded as a superposition of motion caused by its entrainment by large vortices and chaotic wandering relative to the gas or liquid within a region whose size is of order a .

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