

TEMPERATURE GREEN' S FUNCTIONS

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Abstract

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PHYSICS

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TEMPERATURE GREEN' S FUNCTIONS FOR A WEAKLY NONIDEAL BOSE-EIN- STEIN SYSTEM

(Presented by Academician N. N. Bogolyubov on 10 XI 1966)

Earlier in ^(1,2), for a Bose system, a theory of temperature elementary excitations was constructed by analogy with the theory of elementary excitations for a Fermi system with long-range interaction ⁽³⁾; recently Hohenberg ⁽⁴⁾ and Havouré ⁽⁵⁾ have shown that at zero temperature $\theta = 0$ and for a small interaction there can be no special collective branch of the spectrum, the existence of which was discussed in ^(1,2). In the present work we give results obtained by the author which confirm ^(4,5) (within the approach he is developing to the theory of a Bose gas ⁽⁶⁾), as well as, apparently, some new results not known in the literature.

First of all, let us make a small remark concerning the simple secular equation (7) of ⁽¹⁾, which is valid when exchange effects are neglected.* In this equation, in $Q_0(q; E)$ one may put $\varepsilon(p) = E(p)$, $A(p) = E(p)$, $B(p) = 0$, and obtain equation (15) of ⁽¹⁾ without the second term. In ⁽¹⁾ it was shown that such an equation has a phonon root as $v(0) \rightarrow \infty$; we shall show that the root is absent in the weak-coupling theory as $v(0) \rightarrow 0$.

Take the function

$$\text{Re}(q, \varepsilon) = \frac{1}{(2\pi)^3} \int dp (n_p - n_{p+q}) \frac{E(p+q) - E(p)}{-\varepsilon^2 + (E(p+q) - E(p))^2}; \quad (1)$$

where the integral should be understood in the sense of the principal value. For $\varepsilon = 0$ the function is positive and, for small q ,

$$\text{Re}(q, 0) \simeq \frac{m^2 \theta}{2\hbar^2 |q|}; \quad (2)$$

as $\varepsilon \rightarrow \infty$ the function assumes negative values, moreover

$$\text{Re}(q, \varepsilon) \simeq -\frac{2E(q)n}{\varepsilon^2} \left(\frac{\theta}{\theta_0} \right)^{3/2}; \quad (3)$$

at $\varepsilon = E(q) = \hbar^2 q^2 / 2m$ the function undergoes a discontinuity with jump, for small q ,

$$\Delta R \simeq \frac{m^2 \theta}{4\hbar^4} \frac{1}{|q|}. \quad (4)$$

As $q \rightarrow 0$, the value of the function on the left goes to infinity, while the value on the right tends to a certain negative limit,

$$\frac{1}{(2\pi)^2} \frac{2}{\theta} \frac{(2m\theta)^{3/2}}{\hbar^3} \int_0^{+\infty} x^2 dx \left(\frac{e^{x^2}}{(e^{x^2} - 1)^2} - \frac{1}{x^4} \right); \quad (5)$$

the schematic behavior of $\text{Re}(q, \varepsilon)$ for small q is presented in Fig. 1.

The corresponding secular equation can be represented in the form

$$\text{Re}(q, \varepsilon) = -1/v(q); \quad (6)$$

* Exchange effects may be neglected for a small long-range interaction ⁽²⁾; for a small short-range interaction the exchange effects are important ^(4,5).

it is quite clear that (6) has no root as $v(0) \rightarrow 0$, in agreement with ^(4,5), where in fact $v(q) = v(0)$ is assumed. This means that there is no root for a small short-range interaction; for a small long-range interaction a root of the equation may exist.

Let us now consider the secular equation (4) from (2) in the case $\theta \neq 0$. We linearize it in Ξ, H, Γ , and Π ; for Ξ, H we restrict ourselves to diagrams of second order, for Γ —to diagrams of first order, and for Π —to zeroth order. Using the expansions in small v for $A(p), B(p)$ at fixed p , and for $\Xi_2(p, E_{2k}), H_2(p, E_{2k}), \Gamma_1(p, E_{2k})$, and $\Pi_0(p, E_{2k})$ at fixed p, E_{2k} , we obtain the approximate equation

$$\begin{aligned} & E_{2k}^2 + E^2(p) + 2n_0 v(p) E(p) + 2E(p) \frac{1}{(2\pi\hbar)^3} \times \\ & \times \int dp' (v(p-p') - v(p')) n_{p'} + (E_{2k}^2 + E^2(p)) \times \\ & \times \frac{v(p)}{(2\pi\hbar)^3} \int dp' \frac{n_{p-p'} - n_{p'}}{E(p') - E(p-p') + iE_{2k}} + \text{terms higher than } v = 0, \end{aligned} \quad (7)$$

Fig. 1. Schematic depiction of the behavior of the function $\text{Re}(q, \varepsilon)$

which, when the fourth term is neglected, coincides with (15) of (1). Taking $p = \rho x$, $\rho = 2m^{1/2}n_0^{1/2}v^{1/2}(0)$, $E_{2k} = 2n_0v(0)z$, i.e., p, E_{2k} are also small for small v , and using the expansions of $\Xi_2(p, E_{2k})$, $H_2(p, E_{2k})$, $\Gamma_1(p, E_{2k})$, and $\Pi_0(p, E_{2k})$ in small v at fixed x, z , we obtain the equation

$$\begin{aligned}
& 4n_0^2v^2(0)z^2 + 4n_0^2v^2(0)x^2(1+x^2) + \frac{2}{\pi\hbar^3}m^{3/2}n_0^{3/2}v^{5/2}(0)\theta(x^2+1) + \\
& + \frac{2}{\pi^3\hbar^3}m^{3/2}n_0^{3/2}v^{5/2}(0)\theta z^2 \int dx' \frac{1}{e(x')} \left(\frac{x'^2 - 1/2}{e(x')} + \frac{(x-x')^2 - 1/2}{e(x-x')} \right) \times \\
& \times \frac{1}{(e(x-x') + e(x'))^2 + z^2} - \frac{1}{\pi^3\hbar^3}m^{3/2}n_0^{3/2}v^{5/2}(0)\theta z^2 \int dx' \left(\frac{1}{E(x')} - \frac{1}{e(x-x')} \right) \times \\
& \times \left(\frac{x'^2 + 1/2}{e(x')} - \frac{(x-x')^2 + 1/2}{e(x-x')} \right) \frac{1}{(e(x') - e(x-x'))^2 + z^2} \\
& + \frac{2}{\pi^3\hbar^3}m^{3/2}n_0^{3/2}v^{5/2}(0)\theta x^2 \int dx' \times \\
& \times \frac{e(x')e(x-x') - \frac{1}{2}(x'^2 + (x-x')^2) - 3x'^2(x-x')^2 - \frac{1}{4}}{e^2(x')e(x-x')} \frac{e(x') + e(x-x')}{(e(x') + e(x-x'))^2 + z^2} \\
& - \frac{1}{\pi^3\hbar^3}m^{3/2}n_0^{3/2}v^{5/2}(0)\theta x^2 \int dx' \frac{e(x')e(x-x') + \frac{1}{2}(x'^2 + (x-x')^2) + 3x'^2(x-x')^2 + \frac{1}{4}}{e^2(x')e^2(x-x')} \times \\
& \times \frac{(e(x') - e(x-x'))^2}{(e(x') - e(x-x'))^2 + z^2} - \frac{1}{\pi^3\hbar^3}m^{3/2}n_0^{3/2}v^{5/2}(0)\theta x^2 \times \\
& \times \int dx' \frac{e(x')e(x-x') + x'^2(x-x')^2 + \frac{1}{2}(x'^2 + (x-x')^2)}{e^2(x')e(x-x')} \frac{e(x-x') + e(x')}{(e(x-x') + e(x'))^2 + z^2} + \\
& + \frac{1}{2\pi^3\hbar^3}m^{3/2}n_0^{3/2}v^{5/2}(0)\theta \int dx' \frac{e(x')e(x-x') - x'^2(x-x')^2 - \frac{1}{2}(x'^2 + (x-x')^2)}{e(x')e(x-x')} \times \\
& \times \frac{e(x') - e(x-x')}{(e(x') - e(x-x'))^2 + z^2} + \text{terms higher than } v^{5/2} = 0,
\end{aligned} \tag{8}$$

where $e(x) = x(x^2 + 1)^{1/2}$; in deriving (8), from the very beginning (4) from (2) was divided by $1 + \frac{v(q)}{V}\Pi$, which is equivalent to multiplying the linearized equation by $1 - \frac{v(q)}{V}\Pi$, i.e., to adding to it the term $\frac{v(q)}{V}\Pi(E_{2k}^2 + A^2 - B^2)$. In (8) the “gap” terms (which do not vanish at $x = 0, z = 0$) enter virtually; they compensate each other (6).

Putting in (8) $z^2 = -s^2x^2$ and passing to the limit $x \rightarrow 0$, we obtain

$$\begin{aligned}
4n_0^2 v^2(0) s^2 &= 4n_0^2 v^2(0) - \frac{1}{3\pi\hbar^3} m^{3/2} n_0^{3/2} v^{5/2}(0) \theta - \\
&- \frac{1}{\pi^2 \hbar^3} m^{3/2} n_0^{3/2} v^{5/2}(0) \theta \int_0^\infty dx \frac{8x^6 + 12x^4 + \frac{13}{2}x^2 + 1}{(x^2 + 1)^2 (x^2 + \frac{1}{2})^2} \times \\
&\times \left(2 + \frac{\sqrt{x^2 + 1}}{2x^2 + 1} \ln \frac{2x^2 + 1 - \sqrt{x^2 + 1}}{2x^2 + 1 + \sqrt{x^2 + 1}} \right) + \text{terms higher than } v^{5/2}, \quad (9)
\end{aligned}$$

whence for the velocity of sound we have

$$c = \frac{n^{1/2} v^{1/2}(0)}{m^{1/2}} + \frac{mv(0)\theta}{\hbar^3} \alpha + \text{terms higher than } v^2, \quad (10)$$

where α is a certain number, n is the total density. Using formulas (8) and (9) from (7) and the thermodynamic relation $mc^2 = n d\mu/dn$, one can obtain the “thermodynamic” velocity of sound

$$c = \frac{n^{1/2} v^{1/2}(0)}{m^{1/2}} - \frac{1}{4\pi\hbar^3} mv(0)\theta + \text{terms higher than } v^2, \quad (11)$$

which, however, does not coincide with the “microscopic” velocity of sound (10). From (8) one can also obtain the damping of elementary excitations

$$\Gamma(\rho x) = \frac{1}{8\pi\hbar^3} m^{3/2} n_0^{1/2} v^{3/2}(0) \theta (5\pi - 8)x + \text{terms } v^{3/2} \text{ higher than } x + \text{terms higher than } v^{3/2}, \quad (12)$$

where $\rho = 2m^{1/2} n_0^{1/2} v^{1/2}(0)$; the damping at $\theta \neq 0$ is proportional to x , and not to x^5 , as at $\theta = 0$.

We now consider equation (4) from (2) in the case $\theta = 0$. Proceeding similarly to the case $\theta \neq 0$, one can obtain for the velocity of sound

$$c = \frac{n^{1/2} v^{1/2}(0)}{m^{1/2}} - \frac{1}{8\pi^2 \hbar^3} \frac{n^{1/2}}{m^{1/2} v^{1/2}(0)} \int_0^\infty p^2 dp \frac{v^2(p)}{E(p)} + \frac{1}{\pi^2 \hbar^3} mn v^2(0) + \text{terms higher than } v^2; \quad (13)$$

using (6), (7) from (7) and the thermodynamic relation

$$m^2 c^2(n, \theta) = n \partial \mu(n, \theta) / \partial n,$$

one can obtain for c an expression exactly coinciding with (13). For the damping we obtain

$$\Gamma(\rho x) = x^5 \frac{3}{20\pi\hbar^3} m^{3/2} n_0^{3/2} v^{5/2}(0) + \text{terms } v^{5/2} \text{ higher than } x^5 + \text{terms higher than } v^3. \quad (14)$$

Formulas (13), (14) coincide with those obtained in the literature.

For the Green's functions at $\theta = 0$ we obtain

$$TG^{-+} = -TG^{--} = n_0 v(0) - \frac{n_0}{4\pi^2\hbar^3} \int_0^\infty p^2 dp \frac{v^2(p)}{E(p)} + \frac{3}{\pi^2\hbar^3} m^{3/2} n_0^{3/2} v^{5/2}(0) + \\ + \text{terms } v^{5/2}, \text{ vanishing as } x \rightarrow 0 + \text{terms higher than } v^{5/2}, \quad (15)$$

$$T = -s^2 x^2 \left(4n_0^2 v^2(0) + \frac{4}{\pi^2\hbar^3} m^{3/2} n_0^{5/2} v^{7/2}(0) \right) + \\ + x^2 \left(4n_0^2 v^2(0) - \frac{1}{\pi^2\hbar^3} n_0^2 v(0) \int_0^\infty p'^2 dp' \frac{v^2(p')}{E(p')} + \frac{40}{3\pi^2\hbar^3} m^{3/2} n_0^{5/2} v^{7/2}(0) \right) + \\ + \text{terms } v^2, v^3 \text{ and } v^{7/2} \text{ higher than } x^2 + \text{terms higher than } v^{7/2}, \quad (16)$$

where it has been set that $E_{2k}^2 = -4n_0^2 v^2(0) s^2 x^2$ and $\rho = 2m^{1/2} n_0^{1/2} v^{1/2}(0) x$.

The result (15), (16) is consistent with the Gavoret relation¹

$$G^{-+} \simeq \frac{n_0}{n} mc^2 \frac{1}{-E^2 + c^2 p^2}$$

which is valid for small E, p . In the approximation adopted, a similar relation also holds for G^{--} .

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¹J. Gavoret, *Application de la théorie des perturbations à l'étude un liquide de Bose au zéro absolu*, Université de Paris, 1963.

References

Note: Figure translations are in progress. See original paper for figures.

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