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Abstract

Full Text

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THEORY OF ELASTICITY

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ON THE DISSIPATIVE FUNCTION IN THE THEORY OF PLASTIC MEDIA

(Presented by Academician L. I. Sedov on 28 I 1967)

The mathematical theory of plasticity proceeds from the definition of the loading function ⁽¹⁾

$$f(\sigma_{ij}, \varepsilon_{ij}^p, \chi_i, k_i) = 0, \quad (1)$$

where σ_{ij} , ε_{ij}^p are, respectively, the components of stress and plastic strain; χ_i are nonholonomic hardening parameters, k_i are constants.

In what follows we shall omit the index p . Thus, it may be assumed that we are dealing with a model of a rigid-plastic body; extending the results to the case of an elastic-plastic material whose elastic properties do not depend on the plastic ones introduces essentially no changes.

The Mises principle (or Bishop-Hill postulate) may be taken as the basis for defining the relation $\sigma_{ij} - e_{ij}$, where e_{ij} are the components of the rate of plastic strain, $e_{ij} = d\varepsilon_{ij}/dt$. With fixed parameters ε_{ij} , χ_i , alongside the actual stress components σ_{ij} satisfying the loading function, one introduces the set of possible stress components σ_{ij}^* , for which

$$f(\sigma_{ij}^*, \varepsilon_{ij}, \chi_i, k_i) \leq 0. \quad (2)$$

The Mises maximum principle asserts that

$$\sigma_{ij} e_{ij} \geq \sigma_{ij}^* e_{ij}. \quad (3)$$

From equality (2) follow the convexity (nonconcavity) of the loading function and the associated flow law, which determines the orthogonality of the vector e_{ij} to the loading surface in the six-dimensional stress space,

$$e_{ij} = \mu \partial f / \partial \sigma_{ij}, \quad (4)$$

where

$$\mu \geq 0, \quad \text{if } f = 0, \quad d'f = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} \geq 0; \quad (5)$$

$$\mu = 0, \quad \text{if } f < 0 \text{ or } f = 0, \quad d'f \leq 0. \quad (6)$$

The parameters χ_i are determined from the differential relations ⁽¹⁾

$$d\chi_i = A_i d'f. \quad (7)$$

Relations (1), (4)–(7) define the model of a hardening plastic body.

In plastic deformation the work performed by the stresses on the components of plastic strain is irreversible. Let us introduce the dissipative function D , which determines the rate of dissipation of mechanical energy per unit time in a unit volume,

$$D = \sigma_{ij} e_{ij}. \quad (8)$$

The scalar product (8), for fixed parameters ε_{ij} , χ_i , for nonconcave loading surfaces, is completely determined by the giv—

of the vector e_{ij} . Consequently,

$$D = D(e_{ij}, \varepsilon_{ij}, \chi_i, k_i). \quad (9)$$

The dissipative function (9) must be homogeneous of the first degree with respect to the components e_{ij} , since relation (8) must not depend on time. Consequently, $D = (\partial D / \partial e_{ij}) e_{ij}$, and from (8) one can obtain

$$(\partial D / \partial e_{ij} - \sigma_{ij}) e_{ij} = 0, \quad e_{ij} \neq 0. \quad (10)$$

For fixed parameters ε_{ij} , χ_i , let us write relation (8) in total differentials

$$e_{ij} d\sigma_{ij} + \sigma_{ij} de_{ij} = \frac{\partial D}{\partial e_{ij}} de_{ij}. \quad (11)$$

Using the relations of the associated flow law (4), we obtain

$$e_{ij} d\sigma_{ij} = \mu \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} = \mu d'f. \quad (12)$$

For fixed parameters ε_{ij}, χ_i , the change in the stress state occurs along the loading surface; consequently, relation (12) is equal to zero. Then from (11) it follows that

$$\sigma_{ij} = \partial D / \partial e_{ij}. \quad (13)$$

Relation (13) is in complete agreement with (10).

Consequently, if the model of a hardening plastic body is determined by relations (1), (4), then a dissipative function (8), (9) can be determined such that (13) holds.

Let us show that it is possible to construct a theory of plasticity based on the definition of a dissipative function

$$\sigma_{ij} e_{ij} = D = D(e_{ij}, \varepsilon_{ij}, \chi_i, k_i). \quad (14)$$

For fixed parameters ε_{ij}, χ_i , along with the actual components of the strain-rate e_{ij} , introduce the set of possible components of the strain-rate e_{ij}^* , for which

$$D(e_{ij}^*, \varepsilon_{ij}, \chi_i, k_i) \leq D(e_{ij}, \varepsilon_{ij}, \chi_i, k_i). \quad (15)$$

Thus, if in the space of strain rates a certain level surface of the dissipative function D is defined, then the vector of the possible strain rate e_{ij}^* lies inside the volume bounded by this surface.

Introduce a maximum principle analogous to Mises' maximum principle ⁽²⁾

$$\sigma_{ij} e_{ij} \geq \sigma_{ij} e_{ij}^*. \quad (16)$$

From inequality (14) there follows the convexity (nonconcavity) of the dissipative function and the associated loading law

$$\sigma_{ij} = \lambda \partial D / \partial e_{ij}, \quad \lambda = D / \left(\frac{\partial D}{\partial e_{ij}} e_{ij} \right). \quad (17)$$

Specific models are determined by an assumption about the structure of the function D . Suppose that D is a homogeneous function of order m of the components e_{ij} . Then $mD = (\partial D / \partial e_{ij}) e_{ij}$. From (17) it follows that $\lambda = 1/m$.

Suppose that the function D is homogeneous of the first degree in the components e_{ij} ; in this case $\lambda = 1$. In this case relation (15) coincides completely with (13). The derivatives $\partial D / \partial e_{ij}$ are functions homogeneous of degree zero with respect to the components e_{ij} . Consequently, the 6 relations (17) can be regarded as functions of 5 variables, for example e_{ij}/e_{11} .

Assuming the solvability of relations (17) with respect to e_{ij}/e_{11} , as a result of eliminating e_{ij} we obtain a certain finite relation of the form (1), not containing the components of the strain rate. Thus, a loading function is obtained in the usual form.

Let us now show that the components of the strain rate are determined according to the associated flow law (4). Differentiating relation (14), we obtain expression (11). Using (13), from (11) we obtain

$$e_{ij}d\sigma_{ij} = 0. \quad (18)$$

Next, differentiating the obtained relation (1) for fixed ε_{ij}, χ_i , we find

$$\frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} = 0. \quad (19)$$

Relations (18), (19) may be considered at the corresponding values of σ_{ij}, e_{ij} , whence it follows that there exists such a multiplier μ that

$$e_{ij} = \mu \partial f / \partial \sigma_{ij}. \quad (20)$$

Thus, the model of a plastic body can be introduced in equivalent ways: either through the definition of a loading function, or through the definition of a function D homogeneous of first degree with respect to e_{ij} . In both cases the corresponding maximum principle should be formulated.

The loading criterion in the second case is written in the form $D \geq 0$. The parameters χ_i are determined in this case from the differential relations $d\chi_i = D dB_i$, where B_i play a role analogous to A_i in relations (7).

In the case when D is homogeneous of the second degree with respect to e_{ij} , then $\lambda = 1/2$. Relations (17) in this case determine a model of a viscous fluid. The classical model of a viscous fluid will occur as a special case.

As an example, let us consider the function

$$D = \sqrt{\varphi e_{ij} e_{ij}} + c \varepsilon_{ij} e_{ij}, \quad \varphi = \varphi(\varepsilon_{ij}, \chi_i, k_i), \quad c = c(\varepsilon_{ij}, \chi_i, k_i). \quad (21)$$

Function (21) is homogeneous of the first degree with respect to the components e_{ij} . According to (17), for $\lambda = 1$ we obtain

$$\sigma_{ij} = (1/\mu) e_{ij} + c \varepsilon_{ij}, \quad \mu = \sqrt{e_{ij} e_{ij}} / \sqrt{\varphi}. \quad (22)$$

From (22) it follows that

$$e_{ij} = \mu(\sigma_{ij} - c\varepsilon_{ij}). \quad (23)$$

Squaring relations (23), we obtain

$$(\sigma_{ij} - c\varepsilon_{ij})(\sigma_{ij} - c\varepsilon_{ij}) = \varphi. \quad (24)$$

Expressions (24) and (23) determine, respectively, the loading function and the associated flow law for models of a plastic body, whose hardening is achieved by expansion of the loading surface and its displacement as a rigid whole in stress space.

Let us note that relations (21), (22) are consequences of (23), (24), and conversely. For $c = 0$, $\varphi = \text{const}$, the theory of ideal plasticity under the Mises plasticity condition takes place. All the above arguments can be generalized to the case of piecewise-smooth functions D .

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CITED LITERATURE

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Note: Figure translations are in progress. See original paper for figures.

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