

AN OPERATIONAL METHOD FOR DETERMINING SOME INTEGRAL CHARACTERISTICS OF NONSTATIONARY TRANSPORT PROCESSES IN ACTIVE MEDIA

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Abstract

Full Text

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PHYSICS

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AN OPERATIONAL METHOD FOR DETERMINING SOME INTEGRAL CHARACTERISTICS OF NONSTATIONARY TRANSPORT PROCESSES IN ACTIVE MEDIA

(Presented by Academician Ya. B. Zel'dovich on 21 XI 1966)

In ⁽¹⁾ a general operational method was given for solving the equations of nonstationary diffusion in multilayer active media consisting of n plane layers with different diffusion coefficients D_j and "conversion" coefficients k_j ($j = 1, 2, \dots, n$). It is known ^(2,3) that the solution $u(x, t)$ of the equation

$$u'_t = Du''_{xx} - ku, \quad (1)$$

which describes the process of nonstationary diffusion (heat conduction) in a homogeneous active medium linearly interacting with the diffusing substance (respectively, in a medium with heat sources whose power is proportional to the temperature), under zero initial conditions and linear boundary conditions not explicitly dependent on t , is expressed through the solution $v(x, t)$ of the equation

$$v'_t = Dv''_{xx}, \quad (2)$$

which is obtained from (1) by means of the substitution

$$u = ve^{-kt} \quad (3)$$

and is considered under the same boundary conditions. Namely,

$$u(x, t) = k \int_0^t e^{-k\tau} v(x, \tau) d\tau + ve^{-kt}, \quad (4)$$

which can be verified by direct substitution in (1).

The general case considered in ⁽¹⁾, which is not reducible to (4), can also be solved by introducing substitutions of the form (3), which leads to boundary conditions containing exponential functions of the variable t .

As shown in ⁽¹⁾, finding the Laplace transforms of the functions $v_j(x, t)$ in this case also presents no difficulty under various boundary conditions. The greatest difficulties are always associated with the transition to the originals of the solutions. Meanwhile, the classical methods for obtaining formulas describing the properties of some region Ω of the medium under consideration, depending on local values of the solutions $u(x_0, t)$, $x_0 \in \Omega$, consist primarily in finding "general" solutions $u(x, t)$ of prescribed systems of equations and in substituting into these solutions the values $x_0 \in \Omega$ that are of interest to us.

We shall show that there exist questions concerning local or volumetric properties of the media under consideration that can be solved by operational methods without passing to the originals, i.e., without obtaining the general solutions $u_j(x, t)$.

Let at points x_0 of some region Ω of the medium under consideration, generally speaking multilayered, the instantaneous rate of formation of some extensive factor Q (substance, energy, quantity of electricity, etc.) be proportional to the instantaneous value $u(x_0, t)$ (i.e., to the instantaneous concentration of the diffusing substance in the case, if we consid-

is the diffusion process, or to the instantaneous value of the temperature if a heat-transfer process is considered):

$$dQ/dt|_{x=x_0} = Ku(x_0, t). \quad (5)$$

Then, if a finite perturbation is applied to the external boundaries of the medium,

$$u|_{\text{boundary}} = f(t), \quad \text{where} \quad \int_0^\infty f(t) dt < \infty,$$

then in any elementary volume $d\Omega_j$ in the j -th layer of the medium under consideration, the total amount of the factor Q formed as $t \rightarrow \infty$ tends to the value

$$dQ_j = K d\Omega \int_0^\infty u_j(x, t) dt. \quad (6)$$

But, according to (3), the integral on the right-hand side of (6) is, by definition, the Laplace transform of the function $v_j(x, t)$, in which the operational variable s is replaced by the quantity k_j :

$$\int_0^\infty u_j(x, t) dt = \int_0^\infty e^{-k_j t} v(x, t) dt = \bar{v}(x, k_j), \quad (7)$$

i.e. $dQ_j = K \bar{v}(x, k_j) d\Omega$, whence

$$Q_j = K \int_\Omega \bar{v}(x, k_j) d\Omega. \quad (8)$$

In multilayer media, if the volume Ω has a nonempty intersection with several layers, i.e. if

$$\Omega = \sum_j \Omega_j, \quad (9)$$

and in each of the layers the proportionality coefficient K takes the value k_j , we have:

$$Q = \sum_{j=1}^n k_j \int_{\Omega_j} \bar{v}_j(x, k) d\Omega_j, \quad (10)$$

where each of the integrals is taken over the region Ω_j .

The coefficients k_j may depend on the coordinates of the point $M(x, y, z)$, and then, instead of (10), one should use the formula

$$Q = \sum_{j=1}^n \int_{\Omega_j} k_j(M) \bar{v}_j(x, k_j) d\Omega_j. \quad (11)$$

In the simplest case, for Q in the case of a diffusion process one may take the total amount q of the diffusing substance that has undergone conversion (for example, radioactive decay or absorption as a result of a chemical reaction⁴, etc.) in the volume Ω . Then the coefficients k_j will be the actual conversion coefficients k_j , i.e.

$$q(\Omega) = \sum_{j=1}^n k_j \int_{\Omega_j} \bar{v}_j(x, k_j) d\Omega_j. \quad (12)$$

Formulas (10), (11), (12), in combination with the method described in ¹, make it possible, without solving the equations of nonstationary transport, to obtain certain integral characteristics of the process as a whole, pertaining to any region of the medium under consideration, directly in the domain of Laplace transforms.

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Note: Figure translations are in progress. See original paper for figures.

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