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1967

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Abstract

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UDC 521.11+533.9

HYDROMECHANICS

SOME PARTICULAR SOLUTIONS OF THE EQUATIONS DESCRIBING AXISYMMETRIC MOTIONS OF A GRAVITATING RAREFIED PLASMA

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(Presented by Academician L. I. Sedov on 1 X 1966)

Consider axisymmetric nonstationary motions of a rarefied plasma in the presence of a magnetic field and self-gravitation. We shall assume here that the magnetic-field intensity vector $\mathbf{H}$ is perpendicular to the trajectories of the particles. Then the magnetic lines of force may be: 1) either straight lines parallel to the axis of symmetry; 2) or concentric circles with centers on the axis of symmetry; 3) or helical lines.

If dissipative processes are absent, then the equations describing such motions (with allowance for pressure anisotropy) may be written in the form

$$\begin{aligned} \rho \frac{dv_r}{dt} + \frac{\partial}{\partial r}(p_{\perp} + h) + \frac{1}{r} \left[(p_{\perp} - p_{\parallel}) \frac{h_{\varphi}}{h} + 2(h_{\varphi} + Gm\rho) \right] &= 0, \\ \frac{d\rho}{dt} + \rho \left(\frac{\partial v_r}{\partial r} + \frac{v_r}{r} \right) &= 0, \quad \frac{\partial m}{\partial r} - 2\pi\rho r = 0, \\ \frac{d}{dt} \left(\frac{h_z}{\rho^2} \right) &= 0, \quad \frac{d}{dt} \left(\frac{h_{\varphi}}{\rho^2 r} \right) = 0, \quad \frac{d}{dt} \left(\frac{p_{\perp}}{\rho h^{1/2}} \right) = 0, \quad \frac{d}{dt} \left(\frac{p_{\parallel} h}{\rho^3} \right) = 0. \end{aligned} \quad (1)$$

Here r is the distance from the particle to the axis of symmetry; t is time; v_r is the radial velocity; ρ is the density; m is the mass; $h = h_z + h_{\varphi}$; $h_z = H_z^2/8\pi$; $h_{\varphi} = H_{\varphi}^2/8\pi$; H_z and H_{φ} are the axial and transverse components of the magnetic-field intensity vector; G is the gravitational constant; $p_{\perp}$ and $p_{\parallel}$ are the pressures, respectively perpendicular and parallel to the magnetic field (transverse and longitudinal pressures).

In case 1) $h_z \neq 0$, $h_{\varphi} = 0$; in case 2) $h_z = 0$, $h_{\varphi} \neq 0$; in case 3) $h_z \neq 0$, $h_{\varphi} \neq 0$.

In the absence of gravitational forces, equations analogous to equations (1) were obtained earlier in works ^(1,2). In work ⁽²⁾ some exact solutions of system (1) for $G = 0$ were found.

If we assume that the dependence of the radial velocity of a particle on r and t has the form

$$v_r = \frac{r}{\mu(t)} \frac{d\mu}{dt}, \quad (2)$$

where $\mu(t)$ is a function of time (to be determined), then by direct verification it is easy to confirm the existence of the following particular exact solutions of system (1).

I.

$$\begin{aligned} \rho &= \rho_0 \mu^{-2}, & h_z &= [1/2 b_3 \rho_0 \xi^2 + b_2 - P(\xi)] \mu^{-4}, \\ h_\varphi &= 0, & p_\perp &= P(\xi) \mu^{-4}, & p_\parallel &= Q(\xi) \mu^{-2}. \end{aligned} \quad (3)$$

Here $P(\xi)$ and $Q(\xi)$ are arbitrary functions of the Lagrangian coordinate $\xi = r/\mu$. The dependence $\mu(t)$ is found from the differential equation:

$$\left(\frac{d\mu}{dt} \right)^2 = b_3 \mu^{-2} - 2b_1 \ln \mu + b_4 = f_1(\mu); \quad (3')$$

$b_1, \dots, b_4$ are arbitrary constants, $\rho_0 = b_1/2\pi G$.

II.

$$\begin{aligned} \rho &= \frac{R'}{r\mu} = \frac{T'}{r^2}, & h_z &= 0, & h_\varphi &= \frac{1}{r^2} [a_4 + a_5(\xi T - \Pi_1) - 2\pi G R^2], \\ p_\perp &= \frac{1}{r\mu^2} (a_2 + a_3 T), & p_\parallel &= \frac{a_1}{\mu^4} T', \\ \left(\frac{d\mu}{dt} \right)^2 &= 2a_3 \mu^{-1} - a_1 \mu^{-2} - 2a_5 \ln \mu + a_6 = f_2(\mu). \end{aligned} \quad (4)$$

Here $R(\xi)$ is an arbitrary function; $a_1, \dots, a_6$ are arbitrary constants; the functions $\Pi_1(\xi)$ and $T(\xi)$ are related to the function $R(\xi)$ by the relation

$$T' = \xi R' = \Pi_1''$$

(the prime denotes differentiation with respect to ξ).

III.

$$\begin{aligned} \rho &= \frac{R'}{r\mu}, & h_z &= \frac{k^2}{r^2 \mu^2} \Phi(\xi), & h_\varphi &= \frac{1}{r^2} \Phi(\xi), \\ p_\perp &= \frac{c_9}{\alpha} \frac{(\mu^2 + k^2)^{1/2}}{\mu^4} \xi R', & p_\parallel &= \frac{c_3}{\mu^2 (\mu^2 + k^2)} \xi R', \end{aligned}$$

$$\left(\frac{d\mu}{dt}\right)^2 = c_1\mu^{-2} - c_3(\mu^2 + k^2)^{-1} - 2c_5 \ln \mu + c_9\mu^{-2}(\mu^2 + k^2)^{1/2} + \quad (5)$$

$$+ \frac{c_9}{k} \frac{2 + \alpha}{\alpha} \ln \frac{k + (\mu^2 + k^2)^{1/2}}{\mu} + c_{11},$$

$$R(\xi) = b\xi^\alpha - \beta, \quad \Phi(\xi) = A - 2\pi GR^2 + c_5 \int R' \xi^2 d\xi.$$

Here $A = 2\pi G\beta^2$; α can take only two values ($\alpha = 1, \alpha = 2$). In this case $\beta = 0$, $3c_1 = k^2 c_5$, if $\alpha = 1$, and $2c_1 = k^2(c_5 - 4\pi Gb)$, if $\alpha = 2$; $c_1, c_3, c_5, c_9, c_{11}, \beta, b, k^2$ are constants.

Let us note that, for the magnetohydrodynamic equations, solutions analogous to solutions I–III were found by A. G. Kulikovskii ⁽³⁾, I. M. Yavorskaya ⁽⁴⁾, McVittie ⁽⁵⁾, and the author ⁽⁶⁾.

Solutions I–III can be generalized to the case when the plasma particles have, in addition to the radial velocity v_r , also a rotational velocity v_φ , determined from the formula

$$v_\varphi^2 = \xi \chi'(\xi) \mu^{-2}. \quad (6)$$

A similar generalization was made earlier by Yu. P. Ladikov ⁽⁷⁾ for magnetic hydrodynamics with isotropic pressure and by V. P. Korobeinikov ⁽²⁾ for the hydrodynamics of a rarefied plasma without taking gravitational forces into account.

Then, in the case $h_\varphi = 0$, we shall have

$$p_\perp = (P + \rho_0 \chi) \mu^{-4}, \quad (7)$$

where $\chi(\xi)$ is an arbitrary function; the remaining functions depend on r and t according to formulas (3), (3').

In the case $h_z = 0$ we obtain:

$$p_\parallel = (a_1 - \chi'/\xi) T' \mu^{-4}, \quad (8)$$

where $\chi(\xi)$ is an arbitrary function, and the functions $\rho(r, t)$, $p_\perp(r, t)$, $h_\varphi(r, t)$, $\mu(t)$ are determined by formulas (4).

For $h_\varphi \neq 0$, $h_z \neq 0$, and $v_\varphi \neq 0$, solution (5) is valid, in which the constants α and A are arbitrary, while the function $\chi(\xi)$ is found from the formula

$$\chi' = k^2 \left[\left(c_5 - \frac{c_1}{k^2} \right) \xi - 4\pi G \frac{R}{\xi} - \frac{2\Phi(\xi)}{\xi^2 R'} \right].$$

Solutions I–III can be used in some specific problems.

The behavior of the functions $f_1(\mu)$, $f_2(\mu)$, which determine various types of gas motion, has been investigated in detail in works ^(3, 4, 6). From these

It follows from the investigations that in case I a complete expansion of the plasma in the presence of gravitational forces and a magnetic field is impossible. If $b_3 < 0$, then collapse of the plasma toward the axis of symmetry cannot occur. In case II expansion is possible if $a_5 < 0$, and impossible if $a_5 > 0$. If $a_1 > 0$, then the plasma particles cannot contract into an infinitely thin filament.

Received
20 IX 1966

REFERENCES

1. G. Chew, M. L. Goldberger, F. E. Low, Proc. Roy. Soc. A, **236**, 442 (1956), Russian translation: *Problems of Modern Physics*, No. 7, 139 (1957).
2. V. P. Korobeinikov, *Applied Mathematics and Technical Physics*, No. 1, 153 (1962).
3. A. G. Kulikovskii, DAN, **114**, No. 5, 984 (1957).
4. I. M. Yavorskaya, DAN, **114**, No. 5, 988 (1957).
5. G. C. McVittie, Rev. Modern Phys., **30**, No. 3, 1080 (1958).
6. E. V. Ryazanov, PMM, **23**, issue 1, 187 (1957).
7. Yu. P. Ladikov, DAN, **137**, No. 2, 303 (1961).

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