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È. I. Grigolyuk, Ya. I. Burak, Ya. S. Podstrigach

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Abstract

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THEORY OF ELASTICITY

Corresponding Member of the Academy of Sciences of the USSR É. I. GRIGOLYUK,

Ya. I. BURAK, Ya. S. PODSTRIGACH

ON AN EXTREMAL PROBLEM OF THERMOELASTICITY FOR AN INFINITE CYLINDRICAL SHELL

A variational problem is considered for determining axisymmetric temperature fields in an infinite cylindrical shell that ensure a minimum of the functional of the elastic strain energy under prescribed conditions in fixed cross sections. A special case of the obtained solution is investigated as applied to conditions of local heating.

1. Let a free infinite isotropic cylindrical shell of radius R be under the action of an axisymmetric temperature field that is constant over the thickness and vanishes at infinity. The thermoelastic state of the shell is characterized by the circumferential force N and the axial moment M , which are determined through the temperature T and the dimensionless deflection $w_0 = w/R$ by the formulas ^(1,2)

$$N = 2Eh(w_0 - \alpha T), \quad M = -\frac{EhR}{2a^2} \frac{d^2 w_0}{d\xi^2}, \quad (1,1)$$

where $\xi = az/R$; $a^4 = 3(1 - \nu^2)R^2/4h^2$; $2h$ is the shell thickness; E is the modulus of elasticity; ν is Poisson's ratio; α is the linear coefficient of thermal expansion; z is the axial coordinate. The deflection function w_0 must satisfy the resolving equation

$$d^4 w_0 / d\xi^4 + 4(w_0 - \alpha T) = 0 \quad (1,2)$$

and tend to zero as $\xi \rightarrow \infty$.

The elastic energy of the shell, taking (1,1), (1,2) into account, is written in the form

$$K[w_0] = \frac{\pi EhR^2}{8a} \int_{-\infty}^{\infty} \left[\left(\frac{d^4 w_0}{d\xi^4} \right)^2 + 4 \left(\frac{d^2 w_0}{d\xi^2} \right)^2 \right] d\xi, \quad (1,3)$$

i.e., it is a functional defined on the set of functions $w_0 = w_0(\xi)$.

The following variational problem is formulated. Find the extremum of the functional $K[w_0]$ on the set of functions $w_0 = w_0(\xi)$ continuous together with their derivatives up to the third order inclusive and vanishing at infinity, which satisfy, in fixed sections $\xi = \xi_j$ ($j = 1, 2, \dots, n$), the conditions

$$d^{(i)}w(\xi_j)/d\xi^i = w_{ij} \quad (i = 0, 1, 2, 3), \quad (1,4)$$

where w_{ij} are arbitrary numbers that can be determined by prescribing, in the sections $\xi = \xi_j$, numerical values of the problem parameters (deflection, temperature, circumferential force, moment).

The formulated problem is equivalent to the following isoperimetric problem. Find the extremum of the functional $K[w_0]$ on the set of continuous and four-times continuously differentiable functions $w_0 = w_0(\xi)$, on which the singular functionals

$$K_{ij}[w_0] = (-1)^i \int_{-\infty}^{\infty} \delta^{(i)}(\xi - \xi_j) w_0(\xi) d\xi, \quad (1,5)$$

where $\delta^{(i)}(\xi)$ is the i -th derivative of the delta function, take the prescribed values

$$K_{ij}(w_0) = w_{ij}. \quad (1,6)$$

Such a problem reduces to finding the absolute extremum of the functional ⁽³⁾

$$K^*[w_0] = \frac{\pi E h R^2}{8a} \int_{-\infty}^{\infty} \left[\left(\frac{d^4 w_0}{d\xi^4} \right)^2 + 4 \left(\frac{d^2 w_0}{d\xi^2} \right)^2 - 2w_0(\xi) \sum_{i=0}^3 \sum_{j=0}^n \gamma_{ij} \delta^{(i)}(\xi - \xi_j) \right] d\xi. \quad (1,7)$$

Here γ_{ij} are arbitrary constants ensuring satisfaction of conditions (1,4).

The Euler equations for the functional $K^*[w_0]$ give

$$\frac{d^8 w_0}{d\xi^8} + 4 \frac{d^4 w_0}{d\xi^4} = \sum_{i=0}^3 \sum_{j=1}^n \gamma_{ij} \delta^{(i)}(\xi - \xi_j). \quad (1,8)$$

In view of (1,2), from (1,8) we find

$$\frac{d^4 T}{d\xi^4} = \frac{1}{4a} \sum_{i=0}^3 \sum_{j=0}^n \gamma_{ij} \delta^{(i)}(\xi - \xi_j). \quad (1,9)$$

Equations (1,2) and (1,8), or (1,9), together with the conditions at infinity constitute the complete system of relations for determining the extremal temperature distribution and the corresponding displacement $w_0(\xi)$.

2. The solution of equations (1,2) and (1,8), satisfying the conditions at infinity, has the form

$$T(\xi) = \frac{1}{8a} \sum_{j=1}^n \left[\frac{\gamma_{0j}}{6} (\xi - \xi_j)^3 + \frac{\gamma_{1j}}{2} (\xi - \xi_j)^2 + \gamma_{2j} (\xi - \xi_j) + \gamma_{3j} \right] \operatorname{sgn}(\xi - \xi_j); \quad (2,1)$$

$$w_0 = \frac{1}{8} \sum_{j=1}^n \left\{ \left[\frac{\gamma_{0j}}{6} (\xi - \xi_j)^3 + \frac{\gamma_{1j}}{2} (\xi - \xi_j)^2 + \gamma_{2j} (\xi - \xi_j) + \gamma_{3j} \right] \operatorname{sgn}(\xi - \xi_j) + e^{-|\xi - \xi_j|} \left[-\frac{\gamma_{0j}}{4} (\cos(\xi - \xi_j) + \sin |\xi - \xi_j|) + \frac{\gamma_{1j}}{2} \sin(\xi - \xi_j) + \frac{\gamma_{2j}}{2} (\cos(\xi - \xi_j) - \sin |\xi - \xi_j|) - \gamma_{3j} \operatorname{sgn}(\xi - \xi_j) \cos(\xi - \xi_j) \right] \right\}. \quad (2,2)$$

In this case the coefficients γ_{ij} must satisfy the relations

$$\sum_{j=1}^n \gamma_{0j} = 0, \quad \sum_{j=1}^n (\gamma_{0j} \xi_j - \gamma_{1j}) = 0, \quad \sum_{j=1}^n (\gamma_{0j} \xi_j^2 - 2\gamma_{1j} \xi_j + \gamma_{2j}) = 0, \quad \sum_{j=1}^n (\gamma_{0j} \xi_j^3 - 3\gamma_{1j} \xi_j^2 + 6\gamma_{2j} \xi_j - 6\gamma_{3j}) = 0. \quad (2,3)$$

The circumferential force N and the axial moment M , calculated by formulas (1.1) with allowance for (2.1), (2.2), will be

$$N = \frac{Eh}{16} \sum_{j=1}^n \left[-\gamma_{0j} (\cos(\xi - \xi_j) + \sin |\xi - \xi_j|) + 2\gamma_{1j} \sin(\xi - \xi_j) + 2\gamma_{2j} (\cos(\xi - \xi_j) - \sin |\xi - \xi_j|) - 4\gamma_{3j} \operatorname{sgn}(\xi - \xi_j) \cos(\xi - \xi_j) \right] e^{-|\xi - \xi_j|}; \quad (2,4)$$

$$M = -\frac{EhR}{32a^2} \sum_{j=1}^n \left\{ 2\gamma_{0j} |\xi - \xi_j| + \gamma_{0j} (\cos(\xi - \xi_j) - \sin |\xi - \xi_j|) e^{-|\xi - \xi_j|} + 2\gamma_{1j} (1 - e^{-|\xi - \xi_j|} \cos(\xi - \xi_j)) \operatorname{sgn}(\xi - \xi_j) + [2\gamma_{2j} (\cos(\xi - \xi_j) + \sin |\xi - \xi_j|) - 4\gamma_{3j} \operatorname{sgn}(\xi - \xi_j) \cos(\xi - \xi_j)] e^{-|\xi - \xi_j|} \right\}$$

Fig. 1

Figure 1: Fig. 1

Fig. 2

Figure 2: Fig. 2

$$+ \sin |\xi - \xi_j|) - 4\gamma_{3j} \sin(\xi - \xi_j)] e^{-|\xi - \xi_j|} \}. \quad (2.5)$$

From formulas (2.1)–(2.5) it is evident that the experimental temperature distribution T found and the force N are described by piecewise-continuous functions. A distribution of T and N continuous in ξ is obtained by setting $\gamma_{3j} = 0$. If one also requires continuity of the first derivative, it is necessary in addition to set $\gamma_{2j} = 0$.

Let us consider the particular case of a solution as applied to boundary conditions of local heating of the simplest form. Let the zone of local heating of the cylindrical shell be bounded by the sections $\xi = \pm\eta$. The temperature in the end sections ($\xi = \pm\eta$) is equal to zero. In the section $\xi = 0$ the temperature T reaches its maximum value, equal to T_0 .

Fig. 1

Fig. 2

The temperature field (2.1) that is extremal for such a problem, satisfies the condition of symmetry with respect to the section $\xi = 0$ and is continuous together with its first derivative, is the field

$$T = T_0 [2|\xi/\eta|^3 - 3(\xi/\eta)^2 + 1] \quad \text{for } |\xi| \leq \eta, \quad T = 0 \quad \text{for } |\xi| \geq \eta. \quad (2.6)$$

The circumferential force and axial moment corresponding to (2.6) are determined by the formulas

$$\begin{aligned} N = \frac{3Eh\alpha T_0}{\eta^3} [& (\cos(\xi + \eta) + \sin |\xi + \eta|) e^{-|\xi + \eta|} + (\cos(\xi - \eta) + \\ & + \sin |\xi - \eta|) e^{-|\xi - \eta|} - 2(\cos \xi + \sin |\xi|) e^{-|\xi|} + \\ & + \eta (e^{-|\xi + \eta|} \sin(\xi + \eta) - e^{-|\xi - \eta|} \sin(\xi - \eta))] , \end{aligned} \quad (2.7)$$

$$M = \frac{3EhR\alpha T_0}{2a^2\eta^3} \left[2|\xi + \eta| + 2|\xi - \eta| - 4|\xi| \right. \\ \left. + (\cos(\xi + \eta) - \sin|\xi + \eta|)e^{-|\xi + \eta|} + (\cos(\xi - \eta) - \sin|\xi - \eta|)e^{-|\xi - \eta|} \right. \\ \left. - 2(\cos\xi - \sin|\xi|)e^{-|\xi|} \right. \quad (2, 8) \\ \left. - \eta(1 - \cos(\xi + \eta)e^{-|\xi + \eta|}) \operatorname{sgn}(\xi + \eta) + \eta(1 - \cos(\xi - \eta)e^{-|\xi - \eta|}) \operatorname{sgn}(\xi - \eta) \right].$$

In Fig. 1 the graphs of $N^* = N/Eh\alpha T_0$ and $M^* = a^2 M/EhR\alpha T_0$ are presented for $\nu = 0.3$; $R/h = 20$; 40 ; $\eta = a$. In Fig. 2 the graphs are given of the quantity N^* in the section $\xi = 0$ and at the two nearest points $\xi = \xi^{(1)}$ and $\xi = \xi^{(2)}$, where N^* attains an extremal value, for values of η in the range $1 \leq \eta \leq 6$.

The found quantities $\xi^{(1)}$ and $\xi^{(2)}$ for several values of η have the following values.

η	1	2	3	4	5	6
$\xi^{(1)}$	0,951	1,74	2,52	3,40	4,37	5,35
$\xi^{(2)}$	3,27	3,67	4,31	5,12	6,03	6,98

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1. E. I. Grigolyuk, in: *Strength in Mechanical Engineering*, 1951.
2. Ya. S. Pidstrygach, S. Ya. Yarema, *Temperature Stresses in Shells*, 1961.
3. I. M. Gelfand, S. V. Fomin, *Calculus of Variations*, Moscow, 1961.

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