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Abstract

Full Text

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THEORY OF ELASTICITY

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UNSTEADY OSCILLATIONS OF A FREE THREE-LAYER STRIP

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§ 1. It is known ^(1,2) that the problem of flexural vibrations of a three-layer plate reduces, under certain assumptions, to a single partial differential equation with respect to the displacement function $\chi(x, t)$. In the case of an unbounded plate this equation may be written in the form

$$(1 - \vartheta k \nabla^2) \nabla^2 \nabla^2 \chi(x, \tau) + \chi_{\tau\tau}(x, \tau) - k \nabla^2 \chi_{\tau\tau}(x, \tau) = \frac{l^4}{D} q_0(x, \tau), \quad (1,1)$$

where $\nabla^2 = \partial^2 / \partial x^2$; $\chi_{\tau\tau} = \partial^2 \chi / \partial \tau^2$; $q_0(x, \tau)$ is the external load; τ is dimensionless time; ϑ, k are dimensionless parameters characterizing the relative flexural stiffness of the load-bearing layers and the relative shear stiffness of the core; l is a linear quantity; D is the flexural stiffness of the three-layer beam (see ^(1,2)).

The deflection $w(x, \tau)$ is related to the displacement function by the relation

$$w(x, \tau) = l(1 - k \nabla^2) \chi(x, \tau). \quad (1,2)$$

For $\vartheta = 1$, equation (1,1) takes the form

$$-k \left(\nabla^2 \nabla^2 \chi - \frac{1}{k} \chi \right) - k \left(\nabla^2 \chi - \frac{1}{k} \chi \right)_{\tau\tau} = q(x, \tau), \quad (1,3)$$

where

$$q(x, \tau) = \frac{l^4}{D} q_0(x, \tau). \quad (1,4)$$

Let us denote

$$\Phi(x, \tau) = \nabla^2 \chi(x, \tau) + a_0 \chi(x, \tau), \quad (1,5)$$

where

$$a_0 = -1/k. \quad (1,6)$$

Substituting (1,4)–(1,6) into (1,3), we obtain

$$\nabla^2 \nabla^2 \Phi(x, \tau) + \Phi_{\tau\tau}(x, \tau) = a_0 q(x, \tau). \quad (1,7)$$

Applying successively to (1,7) the Fourier and Laplace transforms with respect to the variables x and τ , we obtain

$$\Phi^*(u, s) = \frac{1}{\omega^2 + s^2} \bar{q}^*(u, s) \quad (1,8)$$

(u, s are the transform parameters), where

$$\omega^2 = u^4. \quad (1,9)$$

According to the convolution rule,

$$\bar{\Phi}(u, \tau) = \frac{a_0}{\omega} \int_0^\tau \bar{q}(u, \tau_1) \sin \omega(\tau - \tau_1) d\tau_1. \quad (1,10)$$

The original (1.10) has the form

$$\Phi(x, \tau) = \frac{a_0}{2\pi} \int_{-\infty}^{\infty} e^{-ixu} \left[\frac{1}{\omega} \int_0^\tau \bar{q}(u, \tau_1) \sin \omega(\tau - \tau_1) d\tau_1 \right] du. \quad (1,11)$$

Since

$$\bar{q}(u, \tau) = \int_{-\infty}^{\infty} q(x, \tau) e^{iux} dx, \quad (1,12)$$

expression (1.11) becomes the following:

$$\Phi(x, \tau) = \frac{a_0}{2\pi} \int_0^\tau \int_{-\infty}^{\infty} q(v, \tau_1) dv \int_{-\infty}^{\infty} e^{iu(v-x)} \frac{\sin u^2(\tau - \tau_1)}{u^2} du. \quad (1,13)$$

Transform the right-hand side of (1.13). Denote

$$I(\mu) = \int_{-\infty}^{\infty} e^{i\xi u} \frac{\sin \mu u^2}{u^2} du. \quad (1.14)$$

Compute the partial derivative of $I(\mu)$ with respect to μ :

$$\frac{\partial I(\mu)}{\partial \mu} = \int_{-\infty}^{\infty} e^{i\xi u} \cos \mu u^2 du = \frac{1}{2} \int_{-\infty}^{\infty} e^{i\xi u + i\mu u^2} du + \frac{1}{2} \int_{-\infty}^{\infty} e^{i\xi u - i\mu u^2} du. \quad (1.15)$$

Transform the integral

$$\int_{-\infty}^{\infty} e^{i\xi u + i\mu u^2} du = \int_{-\infty}^{\infty} e^{i\mu(u+\xi/2\mu)^2 - i\xi^2/4\mu} du = \sqrt{\frac{\pi}{|\mu|}} e^{i \cdot 1/4\pi \operatorname{sgn} \mu - i\xi^2/4\mu}. \quad (1.16)$$

Similarly, we find

$$\int_{-\infty}^{\infty} e^{i\xi u - i\mu u^2} du = \sqrt{\frac{\pi}{|\mu|}} e^{-i \cdot 1/4\pi \operatorname{sgn} \mu + i\xi^2/4\mu}. \quad (1.17)$$

Taking (1.16) and (1.17) into account, we write (1.15) in the form

$$\frac{\partial I}{\partial \mu} = \sqrt{\frac{\pi}{|\mu|}} \cos \left(\frac{\pi}{4} - \frac{\xi^2}{4|\mu|} \right). \quad (1.18)$$

Integrating (1.18), we obtain

$$I(\mu) = I(0) + \int_0^\mu \frac{\partial I}{\partial \mu} d\mu. \quad (1.19)$$

Bearing in mind the zero initial conditions, set $I(0) = 0$. Then

$$I(\mu) = \int_0^\mu \sqrt{\frac{\pi}{|\mu|}} \cos \left(\frac{\pi}{4} - \frac{\xi^2}{4|\mu|} \right) d\mu = \operatorname{sgn} \mu \int_0^{|\mu|} \sqrt{\frac{\pi}{u}} \cos \left(\frac{\pi}{4} - \frac{\xi^2}{4u} \right) du. \quad (1.20)$$

Putting $\xi = v - x$, $\mu = \tau - \tau_1$, we obtain

$$\int_{-\infty}^{\infty} e^{iu(v-x)} \frac{\sin u^2(\tau - \tau_1)}{u^2} du = I(\tau - \tau_1) = \int_0^{\tau - \tau_1} \sqrt{\frac{\pi}{u}} \cos \left(\frac{\pi}{4} - \frac{v-x}{4u} \right) du. \quad (1.21)$$

Changing the order of integration in the right-hand side of (1.13) and using (1.20), we find

$$\Phi(x, \tau) = -\frac{a_0}{2\sqrt{\pi}} \int_0^\tau \frac{d\xi}{\sqrt{\xi}} \int_0^{\tau-\xi} d\tau_1 \int_{-\infty}^{\infty} q(v, \tau_1) \cos\left(\frac{\pi}{4} - \frac{v-x}{4\xi}\right) dv. \quad (1.22)$$

It is hardly possible to evaluate the integrals on the right-hand side of (1.22) for an arbitrary function $q(v, \tau)$ and an arbitrary range of variation of the variables. However, some asymptotic estimates are possible.

§ 2. Let the load $q(v, \tau)$ be a unit impulse moving with velocity V in the positive direction of the x -axis. Then

$$\int_{-\infty}^{\infty} q(v, \tau_1) \cos\left(\frac{\pi}{4} - \frac{v-x}{4\xi}\right) dv = \cos\left(\frac{\pi}{4} - \frac{V_*\tau_1 - x}{4\xi}\right), \quad (2.1)$$

where $V_* = V/[2]\sqrt{\Omega/D}$ is the dimensionless velocity of motion of the load. Taking (2.1) into account, we write (1.22) in the form

$$\Phi(x, \tau) = -\frac{a_0}{2\sqrt{\pi}} \int_0^\tau \frac{d\xi}{\sqrt{\xi}} \int_0^{\tau-\xi} \cos\left(\frac{\pi}{4} - \frac{V_*\tau_1 - x}{4\xi}\right) d\tau_1, \quad (2.2)$$

or, evaluating the inner integral,

$$\begin{aligned} \Phi(x, \tau) = & -\frac{2a_0}{V_*\sqrt{\pi}} \int_0^\tau \sqrt{\xi} \sin\left[\frac{\pi}{4} - \frac{V_*(\tau-\xi) - x}{4\xi}\right] d\xi + \\ & + \frac{2a_0}{V_*\sqrt{\pi}} \int_0^\tau \sqrt{\xi} \sin\left(\frac{\pi}{4} + \frac{x}{4\xi}\right) d\xi. \end{aligned} \quad (2.3)$$

The integrands on the right-hand side of (2.2) oscillate strongly near $\xi = 0$. It is obvious, however, that the integrals converge. If (1.22) has been evaluated, then, using (1.6), we find

$$\bar{\Phi}(u, \tau) = (u^2 - b^2)\chi(u, \tau), \quad (2.4)$$

where $b = 1/\sqrt{k}$.

Finally, the displacement function $\chi(x, \tau)$ will be

$$\chi(x, \tau) = \frac{1}{b} \int_0^\tau \Phi(x, \tau_1) \operatorname{sh} b(\tau - \tau_1) d\tau_1, \quad (2.5)$$

where $\Phi(x, \tau)$ is determined according to (2.3).

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REFERENCES

1. E. I. Grigolyuk, P. P. Chulkov, DAN, 149, No. 1 (1963).
2. E. I. Grigolyuk, P. P. Chulkov, Izv. AN SSSR, Mekh. i mashinostr., No. 1 (1964).

Note: Figure translations are in progress. See original paper for figures.

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