

ON CHARACTERISTIC PROPERTIES OF SEISMIC TRAVEL-TIME CURVES

M. L. Gerver, V. M. Markushevich

1967

SovietRxiv

View the original and related papers at <https://sovietrxiv.org/items/ru-196701.02008>

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

UDC 550.34

GEOPHYSICS

M. L. GERBER, V. M. MARKUSHEVICH

ON CHARACTERISTIC PROPERTIES OF SEISMIC TRAVEL-TIME CURVES

(Presented by Academician M. A. Sadovsky, September 19, 1966)

Investigating the inverse problem of geometrical seismics for a spherically symmetric medium with a finite number of waveguides, in papers ⁽¹⁾ and (in more detail) ⁽²⁾ we considered questions of uniqueness in determining a velocity section from travel-time curves of surface and deep sources. In this note we shall formulate conditions under which a solution of the inverse problem exists.

I. 1. Let us recall the formulation of the problem (see Fig. 1a). At a point A on the circumference, at the moment $t = 0$, an impulsive disturbance arises, which propagates inside the disk C along rays according to the laws of geometrical optics with velocity $v(r)$. At every point where the impulse again emerges on the circumference, the arrival time t is recorded.

Fig. 1

Denote by $\tilde{\theta}$ the angular epicentral distance, and by α the angle between the ray and the radius at the source. Then in the plane θ, t we shall have the curve $\tilde{\Gamma}_0\{\theta = \tilde{\theta}(\alpha), t = t(\alpha)\}$. It is required to determine from the curve $\tilde{\Gamma}_0$ the propagation velocity of the impulse $v(r)$, $r \in [0, R]$.

2. Assuming that not only the angle $\tilde{\theta}$ is known, but also the number k of revolutions of the ray about the center of the disk, we consider the curve $\Gamma_0\{\theta = \Theta(\alpha), t = t(\alpha)\}$, $\alpha \in (0, \pi/2)$, where $\Theta(\alpha) = \tilde{\Theta}(\alpha) + 2\pi k$.

3. The transformation

$$x = \frac{R\theta}{v(R)}, \quad y = \frac{R}{v(R)} \ln \frac{R}{r}, \quad u(y) = \frac{v(Re^{-v(R)y/R})}{v(R)e^{-v(R)y/R}}$$

makes it possible to consider a simpler problem of the propagation of an impulse

Fig. 2

Figure 2: Fig. 2

in the half-plane $x, y, y \geq 0$, with velocity $u(y)$, where $u(0) = 1$. The curves $\tilde{\Gamma}_0$ and Γ_0 are then transformed into curves in the x, t plane

$$\tilde{\Gamma}\{x = 2\tilde{X}(p), t = 2T(p)\}, \quad \Gamma\{x = 2X(p), t = 2T(p)\}, \quad p \in (0, 1),$$

where $p = \sin \alpha$ is the ray parameter; $X(p)$ is the abscissa of the deepest point of the ray with parameter p ; $T(p)$ is the time of motion to the deepest point; $\tilde{X}(p) \equiv X(p) \pmod{\pi R/v(R)}$, $0 \leq \tilde{X}(p) < \pi R/v(R)$ (see Fig. 1b).

4. We shall not consider the problem of determining $X(p)$ and $T(p)$ from Γ (see (2)) and of the mapping $\Gamma \rightarrow \tilde{\Gamma}$, but shall deal with questions of existence in determining $u(y)$ from $X(p)$ and $T(p)$.

II. 1. The functions $X(p)$ and $T(p)$ are expressed in terms of $u(y)$ as follows:

$$X(p) = \int_0^{Y(p)} \frac{pu(y) dy}{\sqrt{1 - p^2 u^2(y)}}, \quad T(p) = \int_0^{Y(p)} \frac{dy}{u(y) \sqrt{1 - p^2 u^2(y)}}, \quad p \in (0, 1), \quad (1)$$

where $Y(p) = \inf\{y, pu(y) \geq 1\}$ is the ordinate of the deepest point of the ray with parameter p .

2. With respect to $u(y)$ we shall assume that it is a positive, piecewise twice continuously differentiable function, bounded on every segment of the half-axis $y \in [0, \infty)$ and unbounded on the whole half-axis; $u(0) = 1$.

In addition, let $u(y)$ form only a finite number of waveguides (for the exact definition see in (1) or (2)). For definiteness we shall assume that the first waveguide does not begin immediately at the surface. Figure 2 shows $u(y)$ with two waveguides (j_1 and j_2).

3. Given $X(p)$ and $T(p)$, (1) may be regarded as a system of equations for $u(y)$. We already know (see (1) or (2)) that this system does not have a unique solution. But it may also have no solutions at all. The question arises: what restrictions must be imposed on the functions $X(p)$ and $T(p)$ so that some $u(y)$, satisfying the conditions II.2, would be a solution of system (1).

4. The answer is

Theorem 1. *In order that the curve $\Gamma\{2X(p), 2T(p)\}$, $p \in (0, 1)$, be a travel-time curve from a surface source for a velocity section $u(y)$, satisfying the conditions II.2, it is necessary and sufficient that the following conditions be fulfilled:*

A. The functions $X(p)$ and $T(p)$: 1) are positive; 2) are differentiable almost everywhere; 3) $T'(p) - pX'(p) = 0$ almost everywhere on $(0, 1)$; 4) for all p where $T(p)$ and $X(p)$ are nondifferentiable (except, possibly, for a finite number of them),

$$X(p \pm 0) = X(p) = T(p \pm 0) = T(p) = \infty.$$

B. The function $\tau(p) = T(p) - pX(p)$: 1) decreases monotonically; 2) $\tau(1-0) = 0$; 3) is continuous everywhere, except at the points p_i , $p_1 > p_2 > \dots > p_n$, where it has jumps

$$\sigma_i = \tau(p_i - 0) - \tau(p_i + 0).$$

C. The function

$$\Phi(q) = \frac{2}{\pi} \int_q^1 \frac{X(p) dp}{\sqrt{p^2 - q^2}} :$$

1) is finite for all $q \in (0, 1)$; 2) does not increase; 3) $\Phi(+0) = +\infty$; 4) there exists such a $C > 0$ that everywhere on (p_{k+1}, p_k) , where $\Phi'(q)$ is finite, the inequality

$$\Phi'(q) < -Cq/\sqrt{p_k^2 - q^2}, \quad 1 \leq k \leq n, \quad p_{n+1} = 0;$$

is satisfied; 5) the function $g(y)$, inverse to $\Phi(q)$, is piecewise twice continuously differentiable.

D. The function

$$\tau(p) + \int_p^1 \sqrt{z^2 - p^2} d\Phi(z)$$

is continuously differentiable for $p \neq p_i$, $i = 1, 2, \dots, n$.

5. Let us note some special features of the conditions II.4.

The function $\tau(p) = T(p) - pX(p)$ is given only on the set where $X(p)$ and $T(p)$ are finite. But since this set is dense in $(0, 1)$ and $\tau(p)$ is continuous everywhere, except for a finite number of points where it has jumps, $\tau(p)$ is defined everywhere on $(0, 1)$.

The number of discontinuities of $\tau(p)$ is equal to the number of waveguides.

If a waveguide is located at the surface, then $p_1 = 1$. In this case condition B.2) becomes B.2') $\tau(1-0) = \sigma_1 > 0$.

Fig. 3

Figure 3: Fig. 3

Fig. 4

Figure 4: Fig. 4

6. Instead of the condition D, which is difficult to verify, let us introduce the condition

D'. $X(p) = \infty$ for no more than a countable set of values $p \in (0, 1)$. This condition is not necessary; however, the totality of conditions A, B, C, and D' is sufficient for Γ to be a hodograph.

7. From condition C the following consequence follows:

$$\overline{\lim_{p \rightarrow p^0-0}} X(p) \geq \underline{\lim_{p \rightarrow p^0+0}} X(p) \quad \text{for any } p^0 \in (0, 1).$$

Therefore the curve in Fig. 3 is not a hodograph.

III.1. Let us pass to the hodograph from a deep source. Applying transformation I.3, we may consider the problem immediately in the half-plane. Put

$$f(y) = (\sup\{u(y^0), 0 \leq y^0 \leq y\})^{-1}.$$

Let the depth of the source be $y = d$, and let $f(d-0) = P$, $f(d+0) = Q$; it is clear that $P \geq Q$.

Fig. 3

Fig. 4

The rays going to the right upward from the source give a part of the hodograph $\Gamma_1\{X_1(p), T_1(p)\}$, where

$$X_1(p) = \int_0^d \frac{p u(y) dy}{\sqrt{1 - p^2 u^2(y)}},$$

$$T_1(p) = \int_0^d \frac{dy}{u(y) \sqrt{1 - p^2 u^2(y)}},$$

$$p \in [0, P].$$

If $X_1(Q) < \infty$, then Γ_1 (see Fig. 4) is the arc OI (when $Q = P$) or OJ (when $Q < P$).

2. Introduce the function $H(r) = \text{mes}\{y, y \leq d, u(y) \leq r\}$; then

$$X_1(p) = \int_0^{P^{-1}} \frac{pr dH(r)}{\sqrt{1-p^2r^2}}, \quad T_1(p) = \int_0^{P^{-1}} \frac{dH(r)}{r\sqrt{1-p^2r^2}}, \quad p \in [0, P]. \quad (2)$$

Equations (2) may be regarded as a system of equations with respect to $H(r)$. Obviously, $H(r)$ is a nondecreasing function and $H(0) = 0$.

3. The uniqueness of the solution of (2) is proved in (2). But the question arises: what properties must the functions $X_1(p)$ and $T_1(p)$ have in order that a solution exist in the class of nondecreasing functions. It is easy to see that this question is equivalent to the question of what conditions must be imposed on the curve Γ_1 so that it be part of a hodograph from a deep source. However, the restrictions on the velocity section $u(y)$ in this case are considerably weaker than in II.2. It is assumed that $u(y)$, $y \in (0, d)$, is positive, bounded, and measurable.

4. Introduce the quantities

$$\beta_i = \int_0^1 v^i T_1(vP) dv, \quad i = 1, 2, \dots$$

Define b_i , $i = 1, 2, \dots$, from the triangular system of equations

$$\beta_{2k+1} = \frac{k!}{(2k+1)!} \sum_{i=1}^{k+1} \frac{(2k-i+1)!}{(k-i+1)!} b_i, \quad k = 0, 1, 2, \dots$$

Put $b_0 = T_1(0)/2$.

Lemma. $T_1(p)$ is representable in the form (2) with a nondecreasing function $H(r)$ if and only if the numbers b_i ($i = 0, 1, 2, \dots$) are a sequence of moments for the function

$$\mathcal{H}_1(z) = \int_1^z \frac{Pt}{4\sqrt{t-1}} dH\left(\frac{2\sqrt{t-1}}{Pt}\right), \quad 1 < z < 2,$$

i.e.,

$$b_i = \int_1^2 z^i d\mathcal{H}_1(z). \quad (3)$$

Theorem 2. In order that the curve $\Gamma_1\{X_1(p), T_1(p)\}$, $p \in [0, P]$, be part of the traveltime curve from a deep source, it is necessary and sufficient that:

A. For every m the quadratic forms

$$\sum_0^m b_{i+j} x_i x_j, \quad \sum_0^m (3b_{i+j+1} - 2b_{i+j} - b_{i+j+2}) x_i x_j$$

be nonnegative.

B. The functions $X_1(p)$, $T_1(p)$ be differentiable for $p \in [0, P)$.

C. $T_1'(p) - pX_1'(p) = 0$.

D. $X_1(0) = 0$.

Condition A is equivalent to condition A' of nonnegativity of the forms

$$\sum_0^m (b_{i+j+1} - b_{j+i}) x_i x_j, \quad \sum_0^m (2b_{i+j} - b_{i+j+1}) x_i x^j$$

and expresses the fact (see (3)) that b_i is a sequence of moments³.

5. Theorems 1 and 2 also give us necessary and sufficient conditions in order that: a) a curve in the x, t plane be the traveltime curve of a pulse reflected from a boundary; b) a part Γ_2 of the traveltime curve from a deep source (see Fig. 4) correspond to some section $u(y)$.

Indeed, the traveltime curve of reflected waves is a curve Γ_1 , stretched by a factor of 2 along both axes, for a source situated at the same depth as the reflecting boundary (see (2)).

As for the curve $\Gamma_2\{X_2(p), T_2(p)\}$, $p \in (0, Q)$, then, as shown in (2), the functions $X_d(p) = [X_2(p) - X_1(p)]/2$ and $T_d(p) = [T_2(p) - T_1(p)]/2$ are analogous to $X(p)$ and $T(p)$, if the surface is transferred to the depth $y = d$. Consequently, they must satisfy the conditions II.4 with changes caused by the fact that $p \in (0, Q)$, and not $(0, 1)$.

Institute of Physics of the Earth
named after O. Yu. Schmidt,
Academy of Sciences of the USSR

Received
15 VII 1966

CITED LITERATURE

1. M. L. Gerver, V. M. Markushevich, DAN, **163**, No. 6 (1965).
2. M. L. Gerver, V. M. Markushevich, *Computational Seismology*, No. 3, Transactions of the Institute of Physics of the Earth named after O. Yu.

Schmidt, Academy of Sciences of the USSR, 1967.

3. M. G. Krein, UMN, **6**, 4 (44) (1951).

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.