

ON A PARTICULAR SOLUTION OF THE PROBLEM OF THE ROTATION OF A HEAVY RIGID BODY ABOUT A FIXED POINT

A. I. Dokshevich

1966

SovietRxiv

View the original and related papers at <https://sovietrxiv.org/items/ru-196601.99829>

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

Abstract

Full Text

UDC 531.38

Mechanics

A. I. DOKSHEVICH

ON A PARTICULAR SOLUTION OF THE PROBLEM OF THE ROTATION OF A HEAVY RIGID BODY ABOUT A FIXED POINT

(Presented by Academician P. Ya. Kochina on 19 VII 1965)

The equations of the problem of the rotation of a heavy rigid body about a fixed point, in the notation of [1], have the form

$$\begin{aligned} \dot{x} &= (a_2 - a_1)yz + (b_2y - b_1z)x, \\ \dot{y} &= (a - a_2)zx + (b_1y + b_2z)z - b_2x^2 - \gamma_3, \\ \dot{z} &= (a_1 - a)xy - (b_1y + b_2z)y + b_1x^2 + \gamma_2, \\ \dot{\gamma}_1 &= \gamma_2\omega_3 - \gamma_3\omega_2, \quad \dot{\gamma}_2 = \gamma_3\omega_1 - \gamma_1\omega_3, \quad \dot{\gamma}_3 = \gamma_1\omega_2 - \gamma_2\omega_1, \end{aligned} \quad (1)$$

where

$$\omega_1 = ax + b_1y + b_2z, \quad \omega_2 = a_1y + b_1x, \quad \omega_3 = a_2z + b_2x.$$

The known integrals of these equations are

$$\begin{aligned} \frac{1}{2}(ax^2 + a_1y^2 + a_2z^2) + (b_1y + b_2z)x - \gamma_1 &= h, \\ \gamma_1x + \gamma_2y + \gamma_3z &= m, \\ \gamma_1^2 + \gamma_2^2 + \gamma_3^2 &= e^2. \end{aligned} \quad (2)$$

Let the center of gravity of the body lie on the perpendicular to the circular section of the gyration ellipsoid:

$$a_2 - a_1 = 0, \quad b_2 = 0.$$

We assume that the initial value of the variable x is different from zero and that $b_1 \neq 0$, as a result of which the conditions under which the Hess and Lagrange solutions occur are not fulfilled. Under these assumptions, equations (1) and the integrals (2) can be written as follows:

$$\begin{aligned}
 dx/d\tau &= -zx, \\
 dy/d\tau &= (a_0 - b_0)zx + zy - \gamma, \\
 dz/d\tau &= (b_0 - a_0)xy - y^2 + x^2 + \beta, \\
 d\alpha/d\tau &= -\gamma(b_0y + x) + b_0\beta z, \\
 d\beta/d\tau &= \gamma(a_0x + y) - b_0\alpha z, \\
 d\gamma/d\tau &= \alpha(b_0y + x) - \beta(a_0x + y);
 \end{aligned} \tag{3}$$

$$\begin{aligned}
 \frac{1}{2}(ax^2 + a_1y^2 + a_2z^2) + (b_1y + b_2z)x - \gamma_1 &= h_0, \\
 \alpha x + \beta y + \gamma z &= m_0, \quad \alpha^2 + \beta^2 + \gamma^2 = l_0^2,
 \end{aligned} \tag{4}$$

where

$$\tau = b_1 t, \quad \alpha = \gamma_1/b_1, \quad \beta = \gamma_2/b_1, \quad \gamma = \gamma_3/b_1, \quad a_0 = a/b_1,$$

$$b_0 = a_1/b_1 = a_2/b_1.$$

The system (3) admits the particular solution

$$y = y_1 x + y_2 x^{-1}, \quad z^2 = r_1 x^2 + r_2 x^{-2} + r_0,$$

$$\alpha = \alpha_0 + \alpha_1 x^2, \quad \beta = \beta_0 + \beta_1 x^2, \quad \gamma = \gamma_0 xz.$$

The dependence of the variables of the problem on time is determined by means of the equation

$$dx/dt = -zx,$$

which can be transformed to the form

$$(dx/dt)^2 = -r_1 b_1^2 (x_1^2 - x^2)(x^2 - x_2^2).$$

The constants $y_1, y_2, r_1, r_2, r_0, \alpha_0, \alpha_1, \beta_0, \beta_1, \gamma_0, x_1^2, x_2^2$ are expressed in terms of the parameters a_0, b_0 as follows:

$$3y_1 = b_0 - 2a_0 + \delta, \quad \delta = \pm \sqrt{a_0^2 - a_0 b_0 + b_0^2 + 3},$$

$$3\gamma_0 = -(a_0 + b_0) + 2\delta, \quad \alpha_0 = \pm l_0, \quad \beta_0 = b_0 \alpha_0,$$

$$(4 + b_0^2)\alpha_1 = \gamma_0(3b_0y_1 + a_0b_0 + 2),$$

$$(4 + b_0^2)\beta_1 = \gamma_0[(b_0^2 - 2)y_1 + b_0 - 2a_0],$$

$$y_2 = \frac{\beta_0}{\gamma_0}, \quad r_1 = -\frac{1}{\gamma_0^2}(\alpha_1^2 + \beta_1^2), \quad r_0 = -\frac{2}{\gamma_0^2}(\alpha_0\alpha_1 + \beta_0\beta_1), \quad r_2 = -\frac{\beta_0^2}{\gamma_0^2},$$

$$x_1^2 = \frac{1}{2r_1} \left(-r_0 + \sqrt{r_0^2 - 4r_1r_2} \right), \quad x_2^2 = \frac{1}{2r_1} \left(-r_0 - \sqrt{r_0^2 - 4r_1r_2} \right).$$

The signs of these constants, except for y_1 , can be determined from the inequalities

$$\delta b_0 < 0, \quad \delta \gamma_0 > 0, \quad b_0\alpha_1 > 0, \quad b_0\alpha_0 < 0,$$

$$\beta_0 < 0, \quad \beta_1 > 0, \quad r_1 < 0, \quad r_2 < 0, \quad r_0 > 0, \quad b_0y_2 > 0.$$

Institute of Mechanics and Computing Center
of the Academy of Sciences of the Uzbek SSR

Received
5 VII 1965

REFERENCES

1. P. V. Kharlamov, *Prikl. matem. i mekhanika*, **27**, 4 (1963).

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.