

CURRENT ALGEBRAS AND DISPERSION RELATIONS

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Abstract

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In the study of the properties of form factors, methods based on the algebra of charges and dispersion relations ⁽¹⁾, on the one hand, and on the algebra of current densities ⁽²⁾, on the other, have recently been successfully applied. In the present work we shall show the equivalence of these two approaches, in particular with respect to the so-called sum rule. In fact, we shall consider the more general case of the algebra of Fourier transforms of charge densities, which at zero momentum passes into the ordinary algebra of charges.

Following ⁽¹⁾, let us postulate, for current-like operators A , B , and C , simultaneous commutation relations of the type

$$[A_\lambda(x), B(y)]_{x_0=y_0} = \delta(\mathbf{x} - \mathbf{y}) C_\lambda(x). \quad (1)$$

From this, with the aid of translational invariance and expansion in a complete system of states, it is not difficult to obtain equations for the matrix elements

$$\langle q | C_\lambda(0) | p \rangle = \sum_n (2\pi)^3 \delta(\mathbf{p}_n + \mathbf{k} - \mathbf{q}) \langle q | A_\lambda(0) | n \rangle \langle n | B(0) | p \rangle - \sum_n (2\pi)^3 \delta(\mathbf{p}_n - \mathbf{k} - \mathbf{p}) \langle q | B(0) | n \rangle \langle n | A_\lambda(0) | p \rangle. \quad (2)$$

We shall now show that the use of dispersion relations also leads to the result (2). For this purpose define the operators

$$Q(\mathbf{k}) = \int A_0(\mathbf{x}, 0) e^{-i\mathbf{k} \cdot \mathbf{x}} d^3x;$$

$$Q^r(k) = \int \theta(x_0) \partial_\lambda (A_\lambda(x) e^{ikx}) d^4x;$$

$$Q^a(k) = \int \theta(-x_0) \partial_\lambda (A_\lambda(x) e^{ikx}) d^4x.$$

It is seen that

$$\begin{aligned} Q^r(k) &= -Q(\mathbf{k}) & \text{for } \text{Im } k_0 > 0; \\ Q^a(k) &= Q(\mathbf{k}) & \text{for } \text{Im } k_0 < 0. \end{aligned} \quad (3)$$

On the basis of formulas (1) and (3) one obtains

$$\langle q|C_0(0)|p\rangle = \pm \int \theta(\mp x_0) \partial_\lambda (\langle q|[A_\lambda(x), B(0)]|p\rangle e^{ikx}) d^4x \quad (4)$$

$$\text{for } \pm \text{Im } k_0 < 0.$$

Let us now note that the right-hand side of the equation does not depend on k and in this sense is a trivial analytic function. To obtain non-trivial analytic properties, let us define two auxiliary functions

$$T^{r,a} = \mp \int \theta(\pm x_0) e^{ilx} \partial_\lambda \langle q|[A_\lambda(x), B(0)]|p\rangle e^{ikx} d^4x. \quad (5)$$

In the limit $l \rightarrow 0$, relation (5) goes over into (4). It is convenient to write the expression for $T^{r,a}$ in the form

$$T^{r,a}(l, k) = T_1^{r,a}(l + k) + ik_\lambda T_{2\lambda}^{r,a}(l + k),$$

where

$$T_1^{r,a} = \mp \int \theta(\pm x_0) e^{i(l+k)x} \partial_\lambda \langle q|[A_\lambda(x), B(0)]|p\rangle d^4x, \quad (6)$$

$$T_{2\lambda}^{r,a} = \mp \int \theta(\pm x_0) e^{i(l+k)x} \langle q|[A_\lambda(x), B(0)]|p\rangle d^4x, \quad \pm \text{Im } k_0 > 0.$$

The requirement of local commutativity makes the functions (6) relativistically invariant and permits their analytic continuation into the complex plane of the variable $\nu = \frac{q}{m}(l + k)$, where m is the mass of a particle with momentum q . This leads to dispersion relations for these functions

$$\text{Re } T(\nu) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{\text{Im } T(\nu')}{\nu' - \nu} d\nu', \quad (7)$$

where translational invariance and expansion in a complete system of states give

$$\text{Im } T_1 = \frac{1}{2} \sum_n (2\pi)^4 \delta(l+k+q-p_n) \langle q | (q-p_n)_\lambda A_\lambda(0) | n \rangle \langle n | B(0) | p \rangle - \frac{1}{2} \sum_n (2\pi)^4 \delta(l+k-p+p_n) \langle q | B(0) | n \rangle \langle n | (p_n-q)_\lambda A_\lambda(0) | p \rangle$$

$$\text{Im } T_{2\lambda} = \frac{1}{2i} \sum_n (2\pi)^4 \delta(l+k+q-p_n) \langle q | A_\lambda(0) | n \rangle \langle n | B(0) | p \rangle - \frac{1}{2i} \sum_n (2\pi)^4 \delta(l+k-p+p_n) \langle q | B(0) | n \rangle \langle n | A_\lambda(0) | p \rangle.$$

Substitution of (8) into (7) leads, in the limit $l \rightarrow 0$, to relation (2). Thus, the algebra of charges together with local commutativity gives the same result as the algebra of currents.

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Note: Figure translations are in progress. See original paper for figures.

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