

# ON THE POSSIBILITY OF TAKING THREE-PARTICLE FORCES INTO ACCOUNT IN THE RELATIVISTIC THREE-BODY SCATTERING PROBLEM

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## Abstract

## Full Text

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PHYSICS

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# ON THE POSSIBILITY OF TAKING THREE-PARTICLE FORCES INTO ACCOUNT IN THE RELATIVISTIC THREE-BODY SCATTERING PROBLEM

(Presented by Academician N. N. Bogolyubov, June 29, 1965)

1°. At present the convenience of using L. D. Faddeev' s integral equations <sup>(1)</sup> for studying the nonrelativistic three-body scattering problem is generally recognized. The merits of these equations are also preserved in the case of applying the relativistic generalization obtained by D. Stoyanov and A. N. Tavkhelidze <sup>(2,3)</sup>. In <sup>(4)</sup>, with the aid of this relativistic generalization, a method was indicated for obtaining matrix elements of the amplitudes of three-particle processes; namely, it was shown that these matrix elements have the form

$$T_{ij} = \chi_{(i)}^{0+} M_{ij} \chi_{(j)}^0, \quad (1)$$

where the transition operators  $M_{ij}$  satisfy the equations

$$M_{ij} = \sum_{\alpha \neq i} K_{\alpha} + \sum_{\beta \neq j} M_{i\beta} g_{\beta} K_{\beta}. \quad (2)$$

Here  $g_i$  is the two-particle Green' s function

$$g_i = g_0 + g_0 K_i g_i, \quad (3)$$

$K_{\alpha}$  are the two-body Bethe-Salpeter kernels,  $g_0$  is the free Green' s function, and  $\chi_{(i)}^0$  are solutions of the two-particle Bethe-Salpeter equations for the  $i$ -th two-particle subsystem.

The transition operators  $M_{ij}$  can also be defined by the relations <sup>(4)</sup>

$$g_i M_{ij} g_j = g_i (K_{\Pi} - K_i) g_{\Pi} = g_{\Pi} (K_{\Pi} - K_i) g_i, \quad (4)$$

where  $g_{\Pi}$  is the complete three-particle Green's function (for the case of pair interaction)

$$g_{\Pi} = g_i + g_i(K_{\Pi} - K_i)g_{\Pi}, \quad (5)$$

$$K_{\Pi} = \sum_{\alpha} K_{\alpha}. \quad (6)$$

From relation (4) it is seen that

$$g_i M_{ij} g_j = g_i M_{ik} g_k. \quad (7)$$

Using (7), we have, instead of (2),

$$M_{ij} = (K_{\Pi} - K_i) + M_{ij} g_j (K_{\Pi} - K_j). \quad (8)$$

**2°.** We shall distinguish two types of possible states of a two-particle system—scattering states and bound states. In an arbitrary coordinate system the total energy  $E_i$  of scattering states is equal to  $\sqrt{\mathbf{p}_j^2 + m_j^2} + \sqrt{\mathbf{p}_k^2 + m_k^2}$ , whereas the energy of a bound state is  $E_i^n = \sqrt{(\mathbf{p}_j + \mathbf{p}_k)^2 + \mu_n^2}$ , where  $\mu_n^i$  is the mass of the bound state. Since

for which the inequality  $\mu_n^i < m_j + m_k$  is satisfied, it is obvious that, for identical momenta entering the expressions for  $E_i$  and  $E_i^n$ , we have  $E_i^n < E_i$ . Making use of this circumstance, one can obtain the following expression for the two-particle Green function (5)\* (in Jacobi coordinates)

$$\hat{g}(P_i \bar{x}_i \bar{y}_i) = P(E_i^n) \int f_{P_i \bar{p}_i}(\bar{x}_i) \hat{g}_0(P_i \bar{p}_i) \varphi_{P_i \bar{p}_i}^+(\bar{y}_i) d\bar{p}_i + \sum_n \varphi_{P_i}^n(\bar{x}_i) \omega_{P_i}^{n+}(\bar{y}_i) \delta(P_i^2 - \mu_n^2) \theta(P_{i0}). \quad (9)$$

Here  $P(E_i^n)$  means that the integral is taken in the sense of the principal value at all points corresponding to the energies of the bound states;  $\omega^n$  satisfies the Bethe–Salpeter equation for the wave function of the bound state

$$\omega_{P_i}^n(\bar{x}_i) = \frac{1}{(2\pi)^8} \int \hat{g}_0(P_i \bar{x}_i \bar{u}_i) \hat{K}_i(P_i \bar{u}_i \bar{v}_i) \omega_{P_i}^n(\bar{v}_i) d\bar{u}_i d\bar{v}_i; \quad (10)$$

$\omega^{+n}$  satisfies the corresponding adjoint equation;  $f$  is the Bethe–Salpeter wave function describing the scattering of two particles, and satisfies the equation

$$f_{P_i \bar{p}_i}(\bar{x}_i) = \varphi_{P_i \bar{p}_i}(\bar{x}_i) + \frac{1}{(2\pi)^8} \int \hat{g}_0(P_i \bar{x}_i \bar{u}_i) \hat{K}_i(P_i \bar{u}_i \bar{v}_i) f_{P_i \bar{p}_i}(\bar{v}_i) d\bar{u}_i d\bar{v}_i, \quad (11)$$

$$\varphi_{P_i \bar{p}_i}(\bar{x}_i) = \exp[-iP_i \bar{x}_i]. \quad (12)$$

Introducing the notation

$$\chi_{P_i \bar{p}_i}^{000}(X \tilde{x}_i \bar{x}_i) = \exp[-iPX - i\tilde{p}_i \tilde{x}_i] \varphi_{\frac{M_i}{M} P + \tilde{p}_i, p_i}(\bar{x}_i); \quad (13)$$

$$\chi_{P_i \bar{p}_i}^{00}(X \tilde{x}_i \bar{x}_i) = \exp[-iPX - i\tilde{p}_i \tilde{x}_i] f_{\frac{M_i}{M} P + \tilde{p}_i, p_i}^-(\bar{x}_i); \quad (14)$$

$$\chi_{P_i \bar{p}_i}^{0n}(X \tilde{x}_i \bar{x}_i) = \exp[-iPX - i\tilde{p}_i \tilde{x}_i] \omega_{\frac{M_i}{M} P + \tilde{p}_i}^n(\bar{x}_i); \quad (15)$$

$$M_i = m_j + m_k, \quad M = m_i + m_j + m_k, \quad (16)$$

multiplying (8) on the right and on the left by the corresponding  $\chi$ 's and integrating over the intermediate variables, we obtain, in accordance with (1) and (9), the following systems of equations:

$$\begin{aligned} T_{ii}^{nm}(P \tilde{p}_i \tilde{p}_i) &= \langle K_{\Pi} - K_i \rangle_{ii}^{nm}(P \tilde{p}_i \tilde{p}_i) + \frac{P}{(2\pi)^{12} i} \int T_{i0}^{n0}(P \tilde{p}_i \tilde{p}_i'' \bar{p}_i'') \\ &\times S_i\left(\frac{m_i}{M} P - \tilde{p}_i''\right) \hat{g}_0\left(\frac{M_i}{M} P + \tilde{p}_i'', \bar{p}_i''\right) \langle K_{\Pi} - K_i \rangle_{ii}^{00,m}(P \tilde{p}_i'' \bar{p}_i'' \tilde{p}_i) d\tilde{p}_i'' d\bar{p}_i'' \\ &+ \frac{\sum_{n'}^{n'}}{(2\pi)^{12} i} \int T_{ii}^{nn'}(P \tilde{p}_i \tilde{p}_i'') S_i\left(\frac{m_i}{M} P - \tilde{p}_i''\right) \delta\left[\left(\frac{M_i}{M} P + \tilde{p}_i''\right)^2 - \mu_{n'}^2\right] \\ &\times \theta\left(\frac{M_i}{M} P_0 + \tilde{p}_{i0}''\right) \langle K_{\Pi} - K_i \rangle_{ii}^{n'm}(P \tilde{p}_i'' \tilde{p}_i) d\tilde{p}_i''; \quad (17a) \end{aligned}$$

$$\begin{aligned} T_{i0}^{n0}(P \tilde{p}_i \tilde{p}_i \bar{p}_i) &= \langle K_{\Pi} - K_i \rangle_{ii}^{n0}(P \tilde{p}_i \tilde{p}_i \bar{p}_i) + \frac{1}{(2\pi)^{12} i} P \int T_{i0}^{n0}(P \tilde{p}_i \tilde{p}_i'' \bar{p}_i'') \\ &\times S_i\left(\frac{m_i}{M} P - \tilde{p}_i''\right) \hat{g}_0\left(\frac{M_i}{M} P + \tilde{p}_i'', \bar{p}_i''\right) \langle K_{\Pi} - K_i \rangle_{ii}^{00,0}(P \tilde{p}_i'' \bar{p}_i'' \tilde{p}_i \bar{p}_i) d\tilde{p}_i'' d\bar{p}_i'' \\ &+ \frac{\sum_{n'}^{n'}}{(2\pi)^{12} i} \int T_{ii}^{nn'}(P \tilde{p}_i \tilde{p}_i'') S_i\left(\frac{m_i}{M} P - \tilde{p}_i''\right) \delta\left[\left(\frac{M_i}{M} P + \tilde{p}_i''\right)^2 - \mu_{n'}^2\right] \theta\left(\frac{M_i}{M} P_0 + \tilde{p}_{i0}''\right) \\ &\times \langle K_{\Pi} - K_i \rangle_{ii}^{n'0}(P \tilde{p}_i'' \tilde{p}_i \bar{p}_i) d\tilde{p}_i''. \quad (17) \end{aligned}$$

\* The caret ^ indicates that the quantity is two-particle.

$$\begin{aligned}
T_{ii}^{nm}(P\tilde{p}_i\tilde{p}'_i) = & \langle K_p - K_i \rangle_{ii}^{nm}(P\tilde{p}_i\tilde{p}'_i) + \\
& + \frac{P}{(2\pi)^{12}_i} \int \langle K_p - K_i \rangle_{ii}^{n,00}(P\tilde{p}_i\tilde{p}''_i\tilde{p}'''_i) S_i\left(\frac{m_i}{M}P - \tilde{p}''_i\right) \times \\
& \times \hat{g}_0\left(\frac{M_i}{M}P + \tilde{p}''_i, \tilde{p}'''_i\right) T_{ii}^{0m}(P\tilde{p}''_i\tilde{p}'''_i) d\tilde{p}''_i d\tilde{p}'''_i \\
& + \frac{\sum_{n'}}{(2\pi)^{12}_i} \int \langle K_p - K_i \rangle_{ii}^{nn'}(P\tilde{p}_i\tilde{p}'_i) \times \\
& \times S_i\left(\frac{m_i}{M}P - \tilde{p}''_i\right) \delta\left[\left(\frac{M_i}{M}P + \tilde{p}''_i\right)^2 - \mu_{n'}^2\right] \theta\left(\frac{M_i}{M}P_0 + \tilde{p}''_{i0}\right) T_{ii}^{n'm}(P\tilde{p}''_i\tilde{p}'_i) d\tilde{p}''_i,
\end{aligned} \tag{18a}$$

$$\begin{aligned}
T_{0i}^{0m}(P\tilde{p}_i\tilde{p}'_i) = & \langle K_p - K_i \rangle_{ii}^{0m}(P\tilde{p}_i\tilde{p}'_i) + \\
& + \frac{P}{(2\pi)^{12}_i} \int \langle K_p - K_i \rangle_{ii}^{0,00}(P\tilde{p}_i\tilde{p}''_i\tilde{p}'''_i) S_i\left(\frac{m_i}{M}P - \tilde{p}''_i\right) \times \\
& \times \hat{g}_0\left(\frac{M_i}{M}P + \tilde{p}''_i, \tilde{p}'''_i\right) T_{0i}^{0m}(P\tilde{p}''_i\tilde{p}'''_i) d\tilde{p}''_i d\tilde{p}'''_i \\
& + \frac{\sum_{n'}}{(2\pi)^{12}_i} \int \langle K_p - K_i \rangle_{ii}^{0n'}(P\tilde{p}_i\tilde{p}'_i) \times \\
& \times S_i\left(\frac{m_i}{M}P - \tilde{p}''_i\right) \delta\left[\left(\frac{M_i}{M}P + \tilde{p}''_i\right)^2 - \mu_{n'}^2\right] \theta\left(\frac{M_i}{M}P_0 + \tilde{p}''_{i0}\right) T_{ii}^{n'm}(P\tilde{p}''_i\tilde{p}'_i) d\tilde{p}''_i.
\end{aligned} \tag{18b}$$

In formulas (17)–(18) the function  $S_i$  is the one-particle Green's function, and  $\langle K_p - K_i \rangle_{ii}^{0m}(P\tilde{p}_i\tilde{p}'_i\tilde{p}''_i)$ , etc., denote  $\langle K_p - K_i \rangle$  integrated with the corresponding functions  $\chi$  (5).

Thus, we have obtained two systems of equations: system (17), which relates the scattering amplitudes on a bound state (without its decay, but with a possible transition to another energy state within the same two-particle subsystem)  $T_{ii}^{nm}$  and the scattering amplitudes of three unbound particles with the formation of two-particle bound states  $T_{i0}^{n0}$ , and system (18), which relates the same scattering amplitudes on a bound state  $T_{ii}^{nm}$  and the scattering amplitudes on a bound state with its decay  $T_{0i}^{0m}$ .

3°. All of the preceding discussion referred to the case of pair interaction. At the same time, the inclusion of three-particle forces may prove essential for certain problems involving three particles <sup>(6)</sup>. Therefore we shall include three-particle forces, assuming that the kernel of the three-particle Bethe–Salpeter equation now has the form

$$K = \sum_{\alpha} K_{\alpha} + K_T = K_p + K_T. \quad (19)$$

Let us introduce new transition operators  $W_{ik}$ , defining them by the equations

$$W_{ik} = (K_p - K_i + K_T) + W_{ij}g_j(K_p - K_k + K_T). \quad (20)$$

Representing  $W_{ik}$  in the form

$$W_{ik} = M_{ik} + A_{ik}, \quad (21)$$

we obtain

$$A_{ik} = K_T + M_{ij}g_jK_T + A_{ij}g_j(K_p - K_k + K_T). \quad (22)$$

Hence, using (4) and (5), we have:

$$g_i A_{ik} g_k = g_p K_T g_p + g_i A_{ik} g_k K_T g_p. \quad (23)$$

Putting  $g_i A_{ik} g_k = g_p T g_p$ , we find the equation for  $T$

$$T = K_T + T g_p K_T = K_T + T g_i K_T + T g_i M_{ik} g_k K_T. \quad (24)$$

Again using equations (4) and (5), we obtain from (21)

$$W_{ik} = M_{ik} + T + T g_k M_{kk} + M_{ij} g_j T + M_{ij} g_j T g_k M_{kk}. \quad (25)$$

4°. Now, applying a technique analogous to that described in 2°, one can obtain expressions for the matrix elements of the transition operators with allowance for three-particle forces, assuming that the matrix elements of the transition operators without allowance for three-particle forces  $M_{ik}$  are known (which can be obtained from equations (17)–(18)) and the matrix elements of the amplitude  $T$  (which should be taken from (24)). Because of lack of space these relations are not presented here.

However, one can use the approximation of strong binding inside the two-particle bound state (7). Then instead of (25) and (24) we obtain, respectively,

$$\begin{aligned} \mathcal{J}_{ii}^{nm}(P\tilde{p}_i\tilde{p}_i) &= \langle K_T \rangle_{ii}^{nm}(P\tilde{p}_i\tilde{p}_i) + \frac{\sum_{n'}}{(2\pi)^{13}} \int \frac{d\tilde{p}_i'' \mathcal{T}_{ii}^{nn'}(P\tilde{p}_i\tilde{p}_i'') S_i(\frac{m_i}{M}P - \tilde{p}_i'')}{\frac{M_i}{M}P_0 + \tilde{p}_{i0}'' - \sqrt{\left(\frac{M_i}{M}\mathbf{P} + \tilde{\mathbf{p}}_i''\right)^2 + \mu_{n'}^2} + i\varepsilon} \times \\ &\times \left\{ \langle K_T \rangle_{ii}^{n'm}(P\tilde{p}_i''\tilde{p}_i) + \sum_{n''} \int \frac{d\tilde{p}_i''' T_{ii}^{n'n''}(P\tilde{p}_i''\tilde{p}_i''') S_i(\frac{m_i}{M}P - \tilde{p}_i''') \langle K_T \rangle_{ii}^{n''m}(P\tilde{p}_i'''\tilde{p}_i)}{\frac{M_i}{M}P_0 + \tilde{p}_{i0}''' - \sqrt{\left(\frac{M_i}{M}\mathbf{P} + \tilde{\mathbf{p}}_i'''\right)^2 + \mu_{n''}^2} + i\varepsilon} \right\}; \end{aligned} \quad (26)$$

$$\begin{aligned} T_{ii}^{*nm}(P\tilde{p}_i\tilde{p}_i') &= T_{ii}^{nm}(P\tilde{p}_i\tilde{p}_i') + \mathcal{J}_{ii}^{nm}(P\tilde{p}_i\tilde{p}_i') \\ &+ \sum_{n'} \int \frac{d\tilde{p}_i'' S_i(\frac{m_i}{M}P - \tilde{p}_i'')}{\frac{M_i}{M}P_0 + \tilde{p}_{i0}'' - \sqrt{\left(\frac{M_i}{M}\mathbf{P} + \tilde{\mathbf{p}}_i''\right)^2 + \mu_{n'}^2} + i\varepsilon} \times \\ &\times \left\{ \mathcal{J}_{ii}^{nn'}(P\tilde{p}_i\tilde{p}_i'') T_{ii}^{n'm}(P\tilde{p}_i''\tilde{p}_i') + T_{ii}^{nn'}(P\tilde{p}_i\tilde{p}_i'') \mathcal{J}_{ii}^{n'm}(P\tilde{p}_i''\tilde{p}_i') \right. \\ &\left. + \sum_{n''} \int \frac{T_{ii}^{nn'}(P\tilde{p}_i\tilde{p}_i'') \mathcal{J}_{ii}^{n'n''}(P\tilde{p}_i''\tilde{p}_i''') S_i(\frac{M_i}{M}P - \tilde{p}_i''') T_{ii}^{n''m}(P\tilde{p}_i'''\tilde{p}_i') d\tilde{p}_i'''}{\frac{M_i}{M}P_0 + \tilde{p}_{i0}''' - \sqrt{\left(\frac{M_i}{M}\mathbf{P} + \tilde{\mathbf{p}}_i'''\right)^2 + \mu_{n''}^2} + i\varepsilon} \right\}; \end{aligned} \quad (27)$$

where

$$\mathcal{J}_{ii}^{nm}(P\tilde{p}_i\tilde{p}_i') = \int \chi_{P_{p_i}}^{0n}(X\tilde{x}_i\bar{x}_i) T(X-Y, \tilde{x}_i\bar{x}_i\tilde{y}_i\bar{y}_i) \chi_{P_{p_i'}}^{+0m}(Y\tilde{y}_i\bar{y}_i) dX dY d\bar{x}_i d\bar{y}_i d\tilde{x}_i d\tilde{y}_i; \quad (28)$$

$$T_{ii}^{*nm}(P\tilde{p}_i\tilde{p}_i') = \int \chi_{P_{p_i}}^{0n}(X\tilde{x}_i\bar{x}_i) W_{ii}(X-Y, \tilde{x}_i\bar{x}_i\tilde{y}_i\bar{y}_i) \chi_{P_{p_i'}}^{+0m}(Y\tilde{y}_i\bar{y}_i) dX dY d\bar{x}_i d\bar{y}_i d\tilde{x}_i d\tilde{y}_i. \quad (29)$$

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## CITED LITERATURE

1. L. D. Faddeev, ZhETF, **39**, 1459 (1960).

2. D. Stoyanov, Preprint of the Joint Institute for Nuclear Research, Dubna, R-1777, 1964.
3. D. Stoyanov, A. N. Tavkhelidze, Phys. Lett., **13**, 76 (1964).
4. V. P. Shelest, D. Stoyanov, Phys. Lett., **13**, 253 (1964).
5. D. Stoyanov, V. P. Shelest, Preprint of the Joint Institute for Nuclear Research, Dubna, R-2066, 1965.
6. R. Sawyer, Preprint, Cambridge, Mass., 1965.
7. D. Stoyanov, V. P. Shelest, Preprint, Dubna, E-2108, 1965.

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