

ON THE THEORY OF OPEN RESONATORS FORMED BY PARALLEL DISKS

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Fig. 1

Figure 1: Fig. 1

Abstract

Full Text

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PHYSICS

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ON THE THEORY OF OPEN RESONATORS FORMED BY PARALLEL DISKS

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1. We consider resonators consisting of two disks placed opposite one another (Fig. 1), with $kl \gg 1$, where $k = \omega/c$ is the wave number. The calculation of these resonators is based on the assumption that, at their edge ($r = a$), electromagnetic waves at frequencies close to their critical frequencies are reflected with the reflection coefficient from the open end of a plane waveguide, which for waves E_{0q} and H_{0q} can be written in the form

Fig. 1

$$\begin{aligned} R_E &= -e^{i(\beta' + i\beta''_E)s}, \\ R_H &= -e^{i(\beta' + i\beta''_H)s}, \end{aligned} \quad (1)$$

where $s = \sqrt{2l/k} w$, and w , in our case, is the radial wave number.

The dependence of β' , β''_E , β''_H on the index q is given in Fig. 2, constructed on the basis of (1). Taking into account the difference between β''_E and β''_H , which is significant for small values of q , leads to qualitatively new results.

2. We introduce the functions $f_E(r)$ and $f_H(r)$, proportional respectively to the electric and magnetic azimuthal components of the field:

$$\begin{aligned} f_E(r) &= C J_{m-1}(wr) + D J_{m+1}(wr), \\ f_H(r) &= C J_{m-1}(wr) - D J_{m+1}(wr), \end{aligned} \quad (2)$$

where C and D are arbitrary constants, $J_m(x)$ is the Bessel function, and m is the azimuthal index.

Using expressions (1), we write the impedance boundary conditions at the edge of the resonator:

$$\begin{aligned} f_E(r) + \frac{y_E}{w} \frac{df_E(r)}{dr} &= 0, \\ f_H(r) + \frac{y_H}{w} \frac{df_H(r)}{dr} &= 0, \end{aligned} \quad (3)$$

where (for $|s| \ll 1$)

$$y_E = \frac{\beta' + i\beta_E''}{2} s, \quad y_H = \frac{\beta' + i\beta_H''}{2} s. \quad (4)$$

Substitution of (4) into (2) gives, in the first approximation in s , for $m = 1, 2, \dots$

$$s = \frac{2\nu_{m-1,n}}{M + \beta' + i\beta''} \quad \text{or} \quad s = \frac{2\nu_{m+1,n}}{M + \beta' + i\beta''}, \quad (5)$$

where

$$M = \sqrt{2k/l} a = 2ka/\sqrt{\pi q}; \quad \beta'' = (\beta_E'' + \beta_H'')/2,$$

$v_{m,n}$ is the n -th zero of the Bessel function $J_m(x)$. Expressions (5) give the natural frequencies of the resonator

$$\omega = \frac{\pi c}{l} \left(\frac{q}{2} + p \right), \quad p = \frac{s^2}{4\pi} = p' - ip''. \quad (6)$$

The ratio D/C , according to the order of the formulas in (5), is determined from

$$\frac{D}{C} = -i \frac{wa}{2m} \frac{\beta_E'' - \beta_H''}{4} s \quad \text{or} \quad \frac{C}{D} = i \frac{wa}{2m} \frac{\beta_E'' - \beta_H''}{4} s. \quad (7)$$

For $m = 0$, the natural frequencies are determined by the formulas

$$s = \frac{2v_{1,n}}{M + \beta' + i\beta_E''} \quad \text{or} \quad s = \frac{2v_{1,n}}{M + \beta' + i\beta_H''}. \quad (8)$$

3. For $m = 1, 2, \dots$, the natural oscillations are characterized by three indices m, n, q and also by that Bessel function (J_{m-1} or J_{m+1}) whose roots give the natural frequencies and which, in the first approximation, determines the field of the natural oscillation.

Fig. 2

Therefore we shall denote these oscillations by four indices: $(m, m-1, n, q)$; $(m, m+1, n, q)$.

The derived relations are valid for $|s| \ll 1$. This means that for $q \sim 1$ one must have $ka \gg 1$, $a \gg l$; for $q \gg 1$ it may also be that $l \gg a$. Thus, no restrictions are imposed on the ratio of the diameter of the disks to the distance between them (for each such ratio there exists its own spectrum of high- Q oscillations). As $q \rightarrow \infty$, the parameters β' , β''_E , β''_H coincide with $\beta = 0.824$, and our formulas pass into the formulas of paper (2).

4. When the disks have small transparency, a plane wave incident on the resonator along the z axis excites in the resonator an electromagnetic field containing only the oscillations $(1, 0, n, q)$:

$$E_x = \sum_{n,q} A_{nq} J_0 \left(\frac{v_{0n} r}{a} \right) [e^{iv_q z} - (-1)^q e^{-iv_q z}], \quad (9)$$

$$H_y = \sum_{n,q} B_{nq} J_0 \left(\frac{v_{0n} r}{a} \right) [e^{iv_q z} + (-1)^q e^{-iv_q z}].$$

and the components E_z and H_z , which we do not write out. Here $kl = v_q l + \pi p$, $v_q l = \pi(q/2 + \rho)$, where the parameter ρ is related to the complex reflection coefficient of the disks R by the relation

$$\rho = \frac{i}{2\pi} \ln R = \rho' - i\rho''.$$

The coefficients A_{nq} and B_{nq} are determined by the formulas

$$\begin{aligned} A_{nq} &= e^{-i(k-v_q)l} \frac{iTE_0}{klv_{0n}J_1(v_{0n})} \frac{\omega_{nq}\omega}{\omega^2 - \omega_{nq}^2}, \\ B_{nq} &= e^{-i(k-v_q)l} \frac{iTE_0}{klv_{0n}J_1(v_{0n})} \frac{\omega^2}{\omega^2 - \omega_{nq}^2}, \end{aligned} \quad (10)$$

where T is the transmission coefficient of the disks; the natural frequencies $\omega_{nq} = \omega'_{nq} - i\omega''_{nq}$ are complex; ω is the excitation frequency. The resonant values of the amplitudes (10), when diffraction losses predominate ($p'' \gg \rho''$), are inversely proportional to $v_{0n}^{5/2}$ and $q^{3/2}$, i.e., with increasing indices n and q they decrease rather rapidly. When losses in the disks predominate, the resonant values (10) are inversely proportional to $v_{0n}^{1/2}$ and do not depend on q .

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Note: Figure translations are in progress. See original paper for figures.

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