

ON AN ASYMPTOTIC EXPANSION IN THE CENTRAL LIMIT THEOREM

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Abstract

Full Text

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MATHEMATICS

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ON AN ASYMPTOTIC EXPANSION IN THE CENTRAL LIMIT THEOREM

(Presented by Academician Yu. V. Linnik on 16 IX 1965)

Let $X_1, X_2, \dots$ be a sequence of mutually independent identically distributed random variables with mathematical expectation $m = EX_1$ and positive variance $\sigma^2 = E(X_1 - m)^2$. Introduce the notation

$$F_n(x) = P \left\{ \frac{1}{\sigma\sqrt{n}} \sum_{j=1}^n (X_j - m) < x \right\}, \quad \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^x e^{-t^2/2} dt.$$

Theorem. Suppose that the following conditions are satisfied:

- 1) $E|X_1|^k < \infty$ for some integer $k \geq 3$;
- 2)

$$\lim_{|t| \rightarrow \infty} |Ee^{itX_1}| < 1.$$

Then there exists a function $\varepsilon(n)$, independent of x , such that

$$\lim_{n \rightarrow \infty} \varepsilon(n) = 0$$

and

$$\left| F_n(x) - \Phi(x) - \sum_{\nu=1}^{k-2} \frac{P_\nu(-\Phi)}{n^{\nu/2}} \right| \leq \frac{\varepsilon(n)}{(1 + |x|^k)n^{(k-2)/2}}$$

for all x ($-\infty < x < \infty$). The functions $P_\nu(-\Phi)$ are defined in the same way as in ^(1,4).

This theorem is a refinement of a theorem of Esseen ^(1,2), according to which, under the conditions of the theorem formulated above, the relation

$$F_n(x) - \Phi(x) - \sum_{\nu=1}^{k-2} \frac{P_\nu(-\Phi)}{n^{\nu/2}} = o \left(\frac{1}{n^{(k-2)/2}} \right)$$

holds as $n \rightarrow \infty$, uniformly with respect to x ($-\infty < x < \infty$).

V. V. Petrov ⁽³⁾ earlier found conditions necessary and sufficient for the validity of the relation

$$\left| \frac{d}{dx} \left(F_n(x) - \Phi(x) - \sum_{\nu=1}^{k-2} \frac{P_\nu(-\Phi)}{n^{\nu/2}} \right) \right| \leq \frac{\varepsilon(n)}{(1 + |x|^k) n^{(k-2)/2}}$$

for all x ($-\infty < x < \infty$), where $\varepsilon(n)$ does not depend on x and

$$\lim_{n \rightarrow \infty} \varepsilon(n) = 0.$$

From the theorem stated above there follows directly a series of consequences concerning the global form of integral limit theorems.

Under the conditions of the theorem, the following assertions are valid:

1. For any $p > 1/k$, as $n \rightarrow \infty$ we have

$$\int_{-\infty}^{\infty} \left| F_n(x) - \Phi(x) - \sum_{\nu=1}^{k-2} \frac{P_\nu(-\Phi)}{n^{\nu/2}} \right|^p dx = o\left(\frac{1}{n^{(k-2)p/2}}\right).$$

2. For any $p \geq 1$, as $n \rightarrow \infty$ we have

$$\int_{-\infty}^{\infty} |F_n(x) - \Phi(x)|^p dx = \int_{-\infty}^{\infty} \left| \sum_{\nu=1}^{k-2} \frac{P_\nu(-\Phi)}{n^{\nu/2}} \right|^p dx + o\left(\frac{1}{n^{(k+p-3)/2}}\right).$$

3. For any $p \geq 1$, as $n \rightarrow \infty$ we have

$$\|F_n(x) - \Phi(x)\| = \left\| \sum_{\nu=1}^{k-2} \frac{P_\nu(-\Phi)}{n^{\nu/2}} \right\| + o\left(\frac{1}{n^{(k-2)/2}}\right).$$

Here

$$\|u(x)\| = \left[\int_{-\infty}^{\infty} |u(x)|^p dx \right]^{1/p}$$

for any function $u(x) \in L_p(-\infty, \infty)$.

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Note: Figure translations are in progress. See original paper for figures.

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