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Abstract

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PHYSICS

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ON THE THEORY OF INDUCED TRANSITIONS IN THE RADIATION OF A LUMINOUS ELECTRON

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Let us consider the radiation of an electron moving in a constant and homogeneous magnetic field, occurring under the influence of an incident external electromagnetic wave. As is known ⁽¹⁾, the probability of induced transitions of an electron from the state E_n to the state $E_{n'}$ is given by the expression:

$$W_{nn'} = \frac{2\pi N(\vec{\kappa})e^2c}{\hbar L^3\kappa} \{(\vec{\alpha}^+ \vec{\alpha}) - (\vec{\alpha}^+ \vec{\kappa}^0)(\vec{\alpha} \vec{\kappa}^0)\} \left| \int_0^t e^{-ict(\kappa_{nn'} - \kappa)} dt \right|^2, \quad (1)$$

where $\omega_{nn'} = c\kappa_{nn'} = |E_n - E_{n'}|/\hbar$ is the resonance transition frequency; $N(\vec{\kappa})$ is the number of incident quanta with momentum $\hbar\vec{\kappa} = \hbar\kappa\vec{\kappa}^0$; α_μ is the matrix element of the Dirac matrix, related to the wave functions of the electron in the magnetic field by the relation

$$\alpha_\mu = \int \psi_n^+ e^{-i\vec{\kappa}\vec{x}} \alpha_\mu \psi_n d^3x. \quad (2)$$

In the case of damping, caused in particular by the finite time (on the average equal to $\tau/2$) of the electron's stay at the initial level, the right-hand side of equality (1) can be generalized by introducing the decaying time factor $\exp(-t/\tau)$. Then the integral

$$I = \left| \int_0^t e^{-ict[\kappa_{nn'} - \kappa]} e^{-t/\tau} dt \right|^2 = \frac{1 + e^{-t/\tau} - 2e^{-t/\tau} \cos(\kappa_{nn'} - \kappa)t}{c^2(\kappa_{nn'} - \kappa)^2 + 1/\tau^2} \quad (3)$$

for a time interval $t \gg \tau$ goes over into the expression

$$I = \tau g(\omega) = \tau \frac{\tau}{1 + \tau^2(\omega_{nn'} - \omega)^2}, \quad (4)$$

in which $\omega = c\kappa$ is the frequency of the incident quanta. Therefore, for large time intervals we may introduce the mean probability per unit time of quantum transitions

$$w_{nn'} = 2 \frac{W_{nn'}}{\tau} = \frac{2\pi N(\vec{\kappa})e^2 c}{\hbar \kappa L^3} \{ \vec{\alpha}^+ \vec{\alpha} - (\vec{\alpha}^+ \vec{\kappa}^0)(\vec{\alpha} \vec{\kappa}^0) \} g(\omega). \quad (5)$$

The wave functions of an electron moving in a constant and homogeneous magnetic field and satisfying the Dirac equation were given in our earlier works ((²), see also (¹)). The energy spectrum of the electron is then given by the expression:

$$E_n = c\hbar K_n = c\hbar \sqrt{k_0^2 + 4n\gamma + k_z^2}, \quad (6)$$

in which $k_0 = m_0 c / \hbar$, n is the principal quantum number ($n = 0, 1, 2 \dots$); $\gamma = e_0 H / 2c\hbar$, k_z is the momentum of the electron along the field.

Determining the frequency of the forced resonant transitions, we find

$$\omega_{n,n \mp \nu} = \nu \Omega \frac{m_0 c^2}{E} \left\{ 1 \pm \frac{\beta^2 \nu}{4n} \right\} = \nu \Omega \frac{m_0 c^2}{E} \left\{ 1 \pm \nu \frac{\hbar \Omega}{2m_0 c^2} \left(\frac{m_0 c^2}{E} \right)^2 \right\}. \quad (7)$$

Here ν is the harmonic number; $\Omega = e_0 H / m_0 c$ is the cyclotron frequency. In this formula and in all subsequent calculations we shall retain, in the expansions, terms of order ν/n no higher than the first, which, as is evident from (7), is equivalent to an expansion in Planck's constant $\hbar$.

Let us now suppose that the incident electromagnetic wave is linearly polarized, with the electric-field vector lying in the plane of the electron's orbit of rotation and directed along the radius toward its center (the so-called σ -component ((³), see also (¹), §28). For simplicity we shall also assume that the angle of incidence of the external electromagnetic wave in the spherical coordinate system is $\theta \sim \pi/2$, i.e., the momentum vector also lies in the plane of the electron's orbit of rotation. Then for the transition probability (5) we obtain (see (²))

$$w_{nn'} = \frac{2\pi N e^2 c}{\hbar \chi L^3} \bar{\alpha}_1^+ \bar{\alpha}_1 g(\omega), \quad (8)$$

where

$$\bar{\alpha}_1^+ \bar{\alpha}_1 = \frac{K K' - k_0^2}{4K K'} (I_{n,n'-1}^2 + I_{n-1,n'}^2) - 2 \frac{4\gamma \sqrt{nn'}}{4K K'} I_{n,n'-1} I_{n-1,n'}. \quad (9)$$

In this expression the Laguerre function, for $n > n'$,*

$$I_{nn'}(y) = \frac{1}{\sqrt{n!n'!}} e^{-y/2} y^{(n-n')/2} Q_{n'}^{n-n'}(y) \quad (10)$$

depends on the argument $y = \chi^2/4\gamma$, where χ is the frequency of the incident photon. Expanding the coefficients in formula (9) with respect to ν/n , we obtain

$$\bar{\alpha}_1^+ \bar{\alpha}_1 = \frac{\gamma}{K_n^2} \left\{ n \mp \frac{\nu}{2}(1 - \beta^2) \right\} \left\{ \begin{array}{l} R_{n,n-\nu}^2, \\ R_{n,n+\nu}^2 \end{array} \right. \quad (11)$$

where

$$R_{n,n'} = I_{n,n-1}(y) - I_{n-1,n'}(y). \quad (12)$$

Let us now write the expression for the energy absorbed per unit time by the electron in resonant transitions (the absorption power)

$$P_n = \hbar \omega_{n,n+\nu} w_{n,n+\nu} - \hbar \omega_{n,n-\nu} w_{n,n-\nu} = \frac{4\pi N(\vec{\chi}) \hbar \omega}{L^3} \nu \frac{\Omega^2}{\omega^2} \frac{e^2}{2m_0} \left(\frac{m_0 c^2}{E} \right)^3 \tau \Phi, \quad (13)$$

where

$$\Phi = \frac{[n + \frac{1}{2}\nu(1 - \frac{3}{2}\beta^2)] R_{n,n+\nu}^2}{1 + \tau^2(\omega_{n,n+\nu} - \omega)^2} - \frac{[n - \frac{1}{2}\nu(1 - \frac{3}{2}\beta^2)] R_{n,n-\nu}^2}{1 + \tau^2(\omega_{n,n-\nu} - \omega)^2}. \quad (14)$$

We transform the denominator in the following way:

$$\tau(\omega_{n,n-\nu} - \omega) = \tau(\omega_{n,n+\nu} - \omega) + \tau(\omega_{n,n-\nu} - \omega_{n,n+\nu}) = x + \frac{\nu}{2n} Q \beta^2, \quad (15)$$

where $Q = \omega\tau$ and the quantity $\omega \simeq \nu\Omega m_0 c^2/E$ is close to the resonant frequency.

Next we consider the expression for $R_{n,n\pm\nu}^2$. As was shown earlier (see, for example, (2)), the Laguerre functions $I_{nn'}$ admit a good approxi-

* In the case $n' > n$, we must interchange the indices n and n' and multiply the entire expression by $(-1)^{n-n'}$.

...mation through Bessel functions

$$I_{nn'}(y) \simeq J_\nu(2\sqrt{yn}) - \frac{(\nu-1)\sqrt{4ny}}{4n} J'_\nu(2\sqrt{yn}), \quad (16)$$

where $\nu = n - n'$.

Retaining terms of order ν/n no higher than the first, we find, with the aid of this approximation,

$$R_{n,n\pm\nu}^2 = 4J_\nu'^2(\nu\beta) \left\{ 1 - \frac{1 \mp \nu}{n} \sqrt{yn} \frac{J_\nu''(\nu\beta)}{J_\nu'(\nu\beta)} \right\}, \quad (17)$$

where the second derivative J_ν'' is not difficult to eliminate with the aid of Bessel's equation.

Substituting this entire expansion into formula (14), we find that

$$\Phi = \frac{4\nu J_\nu'^2(\nu\beta)}{1+x^2} \left\{ \frac{\nu(1-\beta^2)}{\beta} \frac{J_\nu(\nu\beta)}{J_\nu'(\nu\beta)} - \frac{3}{2}\beta^2 + \frac{\beta^2 x Q}{1+x^2} \right\}. \quad (18)$$

Finally, the absorbed energy (13) can be expressed in terms of the electric-field strength of the incident wave. Indeed, proceeding from the expression for the energy density $\mathcal{E}^2/4\pi = \hbar\omega N/L^3$, we obtain

$$P_n = \frac{e^2 \mathcal{E}^2 \tau}{2m_0} \frac{m_0 c^2}{\nu E} \Phi. \quad (19)$$

The formula obtained is applicable for any harmonic number ν and has no restrictions associated with the velocity of the electron.

From (18), in the particular case of a nonrelativistic electron $\beta \ll 1$, for dipole transitions, when in (18) one may put

$$J_1(\nu\beta) = \nu\beta/2, \quad J_1'(\nu\beta) = 1/2, \quad E \sim m_0 c^2,$$

we find the result obtained in (4):

$$P_n = \frac{e^2 \mathcal{E}^2 \tau}{2m_0} \frac{1}{1+x^2} \left[1 + \beta^2 \frac{Qx}{1+x^2} \right]. \quad (20)$$

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