

RELATIONSHIP BETWEEN THE STRUCTURE AND POLARIZATION OF PARTICLES IN THE REACTIONS $(l+\tilde{l} \rightarrow p+\tilde{p})$

A. A. Sokolov, B. K. Kerimov, I. G. Dzhafarov, R. Sh. Yakhyaev
1966
SovietRxiv

Abstract

Full Text

UDC 539.124 : 539.125 : 539.126.3

PHYSICS

A. A. SOKOLOV, B. K. KERIMOV, I. G. DZHAFAROV, R. Sh. YAKHYAEV

RELATIONSHIP BETWEEN THE STRUCTURE AND POLARIZATION OF PARTICLES IN THE REACTIONS $l + \tilde{l} \rightleftharpoons p + \tilde{p}$

(Presented by Academician N. N. Bogolyubov, 18 V 1965)

Experiments with colliding beams open up new prospects for testing the limits of applicability of quantum electrodynamics and for studying the structure (form factors) of elementary particles, which is one of the important problems of high-energy physics. When particle-antiparticle beams collide, depending on the conditions, both scattering processes (s -channel) and annihilation processes (t -channel) may occur. As is known, when the s -channel is realized, the form factors of the participating particles depend on spacelike values of the transferred momentum ($q^2 > 0$) and are real. This latter property facilitates their determination. When the t -channel is realized, however, one deals with form factors that are functions of timelike momentum transfers ($q^2 < -4M^2$). In this region the form factors are complex, and direct measurements of the cross section of the annihilation process with unpolarized particles make it possible to determine only the squares of their moduli. For a complete determination of the form factors themselves, experiments with polarized particles are necessary; these were discussed in ⁽¹⁾.

In the present paper we consider the annihilation processes

$$e^-(\mu^-) + e^+(\mu^+) \rightleftharpoons p + \tilde{p}$$

with polarized extended particles. The cross sections of these processes, taking into account the polarization of only one of the strongly interacting particles, were obtained in a number of works ^(1,2,10). In doing so, the authors treated the electron (muon) as a point particle without an anomalous magnetic moment. In the case where the initial and final particles are longitudinally polarized, these processes were studied in detail in ⁽³⁻⁶⁾. The reactions $p\tilde{p} \rightarrow \tilde{l}l$ in the case of arbitrary polarization of both initial particles were considered in ⁽⁷⁾. In the present work it is shown that studying only longitudinal polarizations is insufficient for determining the timelike form factors of particles. For this, experiments with transversely polarized particles are required.

In the one-photon approximation of perturbation theory, the amplitude for the transformation of an electron-positron pair* into a proton-antiproton pair

$$e^-(\mu^-) + e^+(\mu^+) \rightarrow p + \bar{p} \quad (1)$$

has the form**

$$S = (\bar{U}_{e+} \Gamma_\mu U_{e-}) \frac{1}{q^2} (\bar{U}_p T_\mu U_{\bar{p}}), \quad (2)$$

where

$$T_\mu = e \left(F_1(q^2) \gamma_\mu + \frac{\chi_p}{2M} F_2(q^2) \sigma_{\mu\nu} q_\nu \right). \quad (3)$$

* What is said here about annihilation and production of an electron-positron pair is also applicable to the case of a μ -pair.

** The system of units $c = \hbar = 1$ and the metric $a_\mu b_\mu = ab - a_0 b_0$, $a_\mu = (a, ia_0)$ are used in this work.

operator of the proton current. Although at present there are no experimental indications of a finite size of the electron (muon), it is not excluded that the structure of the lepton may manifest itself at the high energies necessary for realization of process (1). In the present work, along with the structure of the proton-antiproton vertex, we also take into account the structures of the electron-positron vertex. To this end we ascribe to the electron a proton-like anomalous magnetic structure, i.e., we choose the electron current in the form

$$\Gamma_\mu = e \left(f_1(q^2) \gamma_\mu - \frac{\chi_e}{2m} f_2(q^2) \sigma_{\mu\nu} q_\nu \right). \quad (4)$$

In the one-photon approximation the form factors include radiative and meson corrections, and it is precisely in this case that it is meaningful to speak of form factors (see, for example, (8)). As for the two-photon approximation of perturbation theory, according to the calculations carried out in (9,10), it changes neither the total cross section nor the angular distribution of the reaction products.

Above, $F_1(q^2)$, $F_2(q^2)$ and $f_1(q^2)$, $f_2(q^2)$ are the Dirac and Pauli form factors ($F_1(0) = F_2(0) = f_1(0) = f_2(0) = 1$); χ_p and χ_e are anomalous magnetic moments; M and m are the masses of the proton and electron (muon), respectively; $q_\nu = k_\nu + k'_\nu = p_\nu + p'_\nu$ is the 4-momentum of the virtual photon; $p_\nu \equiv (\mathbf{p}, iE)$, $p'_\nu \equiv (\mathbf{p}', iE')$, $k_\nu \equiv (\mathbf{k}, iK)$, and $k'_\nu \equiv (\mathbf{k}', iK')$ are, respectively, the 4-momenta of the proton, antiproton, electron, and positron; $\sigma_{\mu\nu} = i(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)/2$, $\gamma_\mu = (\rho_2 \sigma, \rho_3)$ are the Dirac matrices.

A calculation in the center-of-inertia system using formulas (2)-(4) gives the following expression for the differential cross section for the production of arbitrarily polarized extended protons and antiprotons, taking into account longitudinal polarizations and form factors of the annihilating electrons and positrons:

$$\begin{aligned}
 d\sigma_{e \rightarrow p}(s, s', \vec{\zeta}, \vec{\zeta}') = \frac{\alpha^2}{32E^2} \frac{\beta}{\beta_e} \Bigg\{ & a|F_m|^2 + G(\theta)(1 + (\vec{\zeta}\vec{\zeta}')) \\
 & + \left[-2G(\theta) + (a + a_1(1 - 2\cos^2\theta))|F_m|^2 + \frac{2a_1}{\omega} \cos^2\theta \operatorname{Re} F_e F_m^* \right] (\mathbf{p}^0 \vec{\zeta})(\mathbf{p}^0 \vec{\zeta}') \\
 & - a_1|F_m|^2 (\mathbf{k}^0 \vec{\zeta})(\mathbf{k}^0 \vec{\zeta}') + \left(|F_m|^2 - \frac{1}{\omega} \operatorname{Re} F_e F_m^* \right) \\
 & \quad \times \left[a_1((\mathbf{p}^0 \vec{\zeta})(\mathbf{k}^0 \vec{\zeta}') + (\mathbf{p}^0 \vec{\zeta}')(\mathbf{k}^0 \vec{\zeta})) + sa((\mathbf{p}^0 \vec{\zeta}) + (\mathbf{p}^0 \vec{\zeta}')) \right] \cos\theta \\
 & + s \frac{a}{\omega} \operatorname{Re} F_e F_m^* ((\mathbf{k}^0 \vec{\zeta}) + (\mathbf{k}^0 \vec{\zeta}')) \\
 & - \frac{1}{\omega} \operatorname{Im} F_e F_m^* \left[a_1((\mathbf{n} \vec{\zeta}) + (\mathbf{n} \vec{\zeta}')) \cos\theta - sa((\mathbf{p}^0 \vec{\zeta})(\mathbf{n} \vec{\zeta}') + (\mathbf{p}^0 \vec{\zeta}')(\mathbf{n} \vec{\zeta})) \right] \sin\theta \Bigg\} d\Omega, \quad (5)
 \end{aligned}$$

where

$$2G(\theta) = a_1|F_m|^2 \sin^2\theta + \frac{1}{\omega^2}|F_e|^2(a + a_1 \cos^2\theta),$$

$$F_e = F_1(q^2) - \frac{q^2}{4M^2} \varkappa_p F_2(q^2), \quad F_m = F_1(q^2) + \varkappa_p F_2(q^2),$$

$$f_e = f_1(q^2) - \frac{q^2}{4m^2} \varkappa_e f_2(q^2), \quad f_m = f_1(q^2) + \varkappa_e f_2(q^2),$$

$$a = (1 - ss')|f_m|^2, \quad a_1 = -a + (1 + ss') \frac{1}{\omega_0^2} |f_e|^2,$$

$$q^2 = -4E^2, \quad \alpha = e^2/4\pi, \quad \beta = p/E, \quad \beta_e = k/E, \quad \omega = E/M, \quad \omega_0 = E/m.$$

Here $\vec{\zeta}$ and $\vec{\zeta}'$ are unit vectors in the directions of polarization of the proton and antiproton in the coordinate systems in which they are at rest ⁽¹¹⁾; s and s' are the quantum numbers of the helicities of the electron and positron ⁽¹²⁻¹⁴⁾; $\mathbf{n} = \frac{[\mathbf{p}^0 \mathbf{k}^0]}{||[\mathbf{p}^0 \mathbf{k}^0]||}$ is the normal to the reaction plane; θ is the angle between the momenta

of the electron ($k^0 = k/k$) and of the proton ($p^0 = p/p$); $d\Omega = \sin\theta d\theta d\varphi$; F_e, F_m and f_e, f_m are the charge and magnetic form factors of the proton and electron; E is the total energy of the particles.

Averaging over the initial and summing over the final spin states of the particles, and also assuming that $f_1(q^2) = 1$, $f_2(q^2) = 0$ (a point electron without an anomalous magnetic moment), we obtain from (5), as a special case, the result of Ref. ⁽¹⁰⁾. In the case of the production of longitudinally polarized protons and antiprotons, formula (5) goes over into formula (2) of Ref. ⁽³⁾.

The expression for the differential cross section (5) contains terms that depend linearly on the polarizations $\vec{\xi}$ and $\vec{\xi}'$. For a complete determination of F_e and F_m , it is important to study these terms. In this case the annihilating ultrarelativistic electrons and positrons ($\beta_e \sim 1$) must be either unpolarized or parallel-longitudinally polarized ($s = -s' = 1$ or -1). As shown in ⁽³⁾, it is precisely these cases that are predominantly realized. In the annihilation of antiparallel-longitudinally polarized ultrarelativistic electrons and positrons ($s = s' = 1$ or -1), the terms linearly dependent on the polarizations $\vec{\xi}$ and $\vec{\xi}'$ vanish.

As is seen from (5), in order to obtain new information about the electron (muon) form factors it is important to study the annihilation of transversely polarized electron-positron pairs. This became possible after it was found ⁽¹⁵⁾ that electrons and positrons moving with ultrarelativistic velocities, under conditions close to those in real storage rings, acquire, as a result of synchrotron radiation, a predominantly transverse polarization: the spins of the electrons are directed opposite to the magnetic field, and the spins of the positrons along the field.

According to formulas (2)–(4) we obtain the following expression for the differential cross section for the annihilation of arbitrarily polarized extended electrons and positrons, leading to the production of longitudinally polarized proton-antiproton pairs (in the c.m.s.):

$$d\sigma_{e \rightarrow p}(s, s', \zeta, \zeta') = \frac{\alpha^2}{64E^2} \frac{\beta}{\beta_e} \left\{ |F_m|^2 (1 - \zeta\zeta') [2|f_m|^2 + (1 + (ss')) \left(\frac{1}{\omega_0^2} |f_e|^2 - b \right) + 2|f_m|^2 (p^0 s)(p^0 s') + +2 \right\}$$

where (ζ and $\zeta' = \pm 1$)

$$b = |f_m|^2 \sin^2 \theta + \frac{1}{\omega_0^2} |f_e|^2 \cos^2 \theta, \quad b_1 = \left(|f_m|^2 + \frac{1}{\omega_0^2} |f_e|^2 - \frac{2}{\omega_0} \operatorname{Re} f_e f_m^* \right) \cos^2 \theta,$$

$$b_2 = 2 \left(|f_m|^2 - \frac{1}{\omega_0} \operatorname{Re} f_e f_m^* \right) ((p^0 s)(k^0 s') + (p^0 s')(k^0 s)) \cos \theta + \frac{1}{\omega_0} \operatorname{Im} f_e f_m^* ((ns) + (ns')) \sin 2\theta.$$

Setting in (6) $(\mathbf{k}^0 \mathbf{s}) = (\mathbf{k}^0 \mathbf{s}') = 0$ and $(\mathbf{s} \mathbf{s}') = -1$, we obtain the annihilation cross section for transversely antiparallel-polarized electrons and positrons. Formula (6) contains both the sine and the cosine of the relative phase of the electron form factors f_e and f_m . If in (6) we set $f_1(q^2) = 1$, $f_2(q^2) = 0$ and sum over the polarizations of the produced protons and antiprotons, then we find the expression for the cross section obtained in (16).

Upon integration over the directions of emission of the proton, the phase terms drop out, and (6) becomes

$$\sigma_{e \rightarrow p}(\mathbf{s}, \mathbf{s}', \zeta, \zeta') = \frac{\pi \alpha^2}{48 E^2} \frac{\beta}{\beta_e} \left(2|F_m|^2(1 - \zeta \zeta') + \frac{1}{\omega^2} |F_e|^2(1 + \zeta \zeta') \right) \times \times \left[2|f_m|^2 + \frac{1}{\omega_0^2} |f_e|^2(1 + (\mathbf{s} \mathbf{s}')) + 2 \left(|f_m|^2 - \right. \right.$$

It follows from the above discussion that, in order to determine form factors, one must study the polarization precisely of those particles whose structure is of interest. It should be noted that, because of the difficulty of studying the polarizations of final particles, for the study of proton form factors process (1) is less advantageous than the process of annihilation of arbitrarily polarized protons and antiprotons into a lepton-antilepton pair:

$$p + \tilde{p} \rightarrow e^- + e^+, \quad p + \tilde{p} \rightarrow \mu^- + \mu^+. \quad (8)$$

The cross sections of these processes can be obtained from formula (5) by means of the relation

$$d\sigma_{p \rightarrow e}(\vec{\zeta}, \vec{\zeta}', \mathbf{s}, \mathbf{s}') = (\beta_e |\beta|)^2 \left\{ d\sigma_{e \rightarrow p}(\mathbf{s}, \mathbf{s}', \vec{\zeta}, \vec{\zeta}') \right\}_{n \rightarrow -n}. \quad (9)$$

Summing the last formula over the spin states of the electrons and positrons, with $f_1(q^2) = 1$ and $f_2(q^2) = 0$, we obtain, in particular, the result of work (7). The same relation as (9) holds between the annihilation cross section for longitudinally polarized protons and antiprotons with production of arbitrarily polarized leptons $d\sigma_{p \rightarrow e}(\zeta, \zeta', \mathbf{s}, \mathbf{s}')$ and the cross section of the inverse reaction $d\sigma_{e \rightarrow p}(\mathbf{s}, \mathbf{s}', \zeta, \zeta')$, determined from formula (6).

In conclusion we note that the formulas obtained above can also be applied to processes of the type

$$e^-(\mu^-) + e^+(\mu^+) \rightarrow \mu^-(e^-) + \mu^+(e^+), \quad n + \tilde{n}$$

and so on.

Moscow State University
named after M. V. Lomonosov

Received
18 V 1965

REFERENCES

1. A. Zichichi, S. M. Berman et al., *Nuovo Cim.*, **24**, 170 (1962).
2. K. I. Barnes, *Nuovo Cim.*, **28**, 284 (1963).
3. F. S. Sadykhov, I. G. Dzhaferov, R. Sh. Yakhyayev, *ZhETF*, **46**, 1463 (1964).
4. I. G. Dzhaferov, *Izv. vyssh. uchebn. zaved., Fizika*, No. 6, 160 (1964).
5. A. A. Sokolov, B. K. Kerimov et al., *DAN*, **161**, No. 6 (1965).
6. R. Sh. Yakhyayev, *Vestn. MGU*, No. 6, 49 (1965).
7. S. M. Bilen' kii, R. M. Ryndin, *Nuclear Physics*, **1**, 84 (1965).
8. V. N. Baier, *ZhETF*, **37**, 1940 (1959); *UFN*, **78**, 619 (1962).
9. A. A. Nikishov, *ZhETF*, **39**, 757 (1960).
10. N. Cabibbo, R. Gatto, *Phys. Rev.*, **124**, 1577 (1961).
11. H. A. Tolhoek, *Rev. Mod. Phys.*, **28**, 277 (1956); A. A. Sokolov, M. M. Kolesnikova, *ZhETF*, **38**, 1778 (1960).
12. A. A. Sokolov, B. K. Kerimov, *Ann. d. Phys.*, **2**, 46 (1958).
13. A. A. Sokolov, *Introduction to Quantum Electrodynamics*, Moscow, 1958.
14. A. A. Sokolov, A. I. Mukhtarov, *Vestn. MGU*, No. 8, 63 (1948).
15. A. A. Sokolov, I. M. Ternov, *DAN*, **153**, 1052 (1963); I. M. Ternov, V. Bagrov, R. A. Rzaev, *Vestn. MGU*, No. 4, 62 (1964).
16. V. N. Baier, V. S. Fadin, *DAN*, **161**, 74 (1965).

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.