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M. P. Mikhailova

1966

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Abstract

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UDC 533.6.011.72

AERODYNAMICS

M. P. MIKHAILOVA

MOTION OF A PISTON IN A HEAT-CONDUCTING AND VISCOUS MEDIUM

(Presented by Academician L. I. Sedov on 22 X 1965)

Consider the one-dimensional motion of a gas ahead of a piston that moves with constant velocity U in a heat-conducting and viscous medium. The basic equations of one-dimensional unsteady motion of a viscous and heat-conducting medium have the form

$$\begin{aligned}\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u)}{\partial r} + (\nu - 1) \frac{\rho u}{r} &= 0, \\ \frac{\partial(r^{\nu-1} \rho u)}{\partial r} + \frac{\partial}{\partial r} [r^{\nu-1} (\rho u^2 - p_{rr})] + (\nu - 1) r^{\nu-2} p_{nn} &= 0, \\ \frac{\partial}{\partial t} \left[\rho r^{\nu-1} \left(\frac{u^2}{2} + \varepsilon \right) \right] + \frac{\partial}{\partial r} r^{\nu-1} \left[\rho u \left(\frac{u^2}{2} + \varepsilon \right) - p_{rr} u - \chi \frac{\partial T}{\partial r} \right] &= 0, \quad (1) \\ p_{rr} &= -p + \lambda \left[\frac{\partial u}{\partial r} + (\nu - 1) \frac{u}{r} \right] + 2\mu \frac{\partial u}{\partial r}, \\ p_{nn} &= -p + \lambda \left[\frac{\partial u}{\partial r} + (\nu - 1) \frac{u}{r} \right] + 2\mu \frac{u}{r},\end{aligned}$$

where t is time; r is the coordinate; ρ is density; u is the velocity component; ε is internal energy; T is temperature; χ is the coefficient of heat conductivity; p is pressure; p_{rr}, p_{nn} are stress coordinates; λ, μ are viscosity coefficients; $\nu = 1, 2, 3$, respectively, for plane, cylindrical, and spherical pistons. Suppose that $p = R\rho T$ and $\varepsilon = C_v T$, where R is the gas constant and C_v is the specific heat conductivity.

Instead of the dimensional independent variables t and r , take the dimensionless independent variables

$$\xi = \frac{\gamma p_1}{\rho_1} \frac{t^2}{r^2} \quad \text{and} \quad \eta = \frac{\chi t}{\rho_1 C_v r^2},$$

where ρ_1, p_1 are the density and pressure ahead of the shock wave. We also introduce the dimensionless constants $q_1 = 2\mu C_v/\chi$ and $q_2 = \lambda C_v/\chi$. The sought dimensional functions are represented in terms of dimensionless functions depending on the dimensionless variables by

$$u = \frac{r}{t} V_1(\xi, \eta), \quad p = \rho_1 \frac{r^2}{t^2} P_1(\xi, \eta), \quad \rho = \rho_1 R_1(\xi, \eta). \quad (2)$$

Let $V_0(\xi), P_0(\xi), R_0(\xi)$ be the solutions of the problem for self-similar motion obtained by L. I. Sedov ⁽¹⁾. Then the required linearized solutions can be represented in the form

$$V_1 = V_0(\xi) + \eta V(\xi), \quad P_1 = P_0(\xi) + \eta P(\xi), \quad R_1 = R_0(\xi) + \eta R(\xi). \quad (3)$$

After passing to dimensionless variables and varying with respect to η , we obtain a system of ordinary differential equations for V, P, R . Assuming that the velocity of the piston is large and that ξ^2 may be neglected, the system takes the form

$$2\xi(1-V_0) dR/d\xi - 2\xi R_0 dV/d\xi + [(\nu-2)R_0 - 2\xi dR_0/d\xi] V + [(\nu-2)V_0 + 1 - 2\xi dV_0/d\xi] R = 0,$$

$$\begin{aligned} & 2\xi(1-2V_0)R_0 dV/d\xi + 2\xi V_0(1-V_0) dR/d\xi - 2\xi dP/d\xi \\ & + 2\{\xi(1-2V_0) dR_0/d\xi + [(\nu-1)V_0 - 2\xi dV_0/d\xi] R_0\} V \\ & + [2\xi(1-2V_0) dV_0/d\xi + \nu V_0^2] R + 2\xi(\nu-2)(q_1 + 2q_2) dV_0/d\xi = 0, \end{aligned} \quad (4)$$

$$\begin{aligned} & \left[V_0 R_0 (2 - 3V_0) - \frac{2\gamma P_0}{\gamma - 1} \right] \xi \frac{dV}{d\xi} + 2\xi \frac{1 - \gamma V_0}{\gamma - 1} \frac{dP}{d\xi} + \xi V_0^2 (1 - V_0) \frac{dR}{d\xi} + \\ & + \left[2\xi V_0 \left(1 - \frac{3}{2} V_0 \right) \frac{dR_0}{d\xi} + 2\xi R_0 (1 - 3V_0) \frac{dV_0}{d\xi} - V_0 R_0 + \frac{3}{2} \nu V_0^2 R_0 + \right. \\ & \quad \left. + \frac{\nu\gamma}{\gamma - 1} P_0 - \frac{2\xi\gamma}{\gamma - 1} \frac{dP_0}{d\xi} \right] V + \frac{1}{\gamma - 1} \left(\nu\gamma V_0 - 1 - 2\xi\gamma \frac{dV_0}{d\xi} \right) P + \\ & + \left[\frac{V_0^2}{2} (\nu V_0 - 1) + 2\xi V_0 \left(1 - \frac{3}{2} V_0 \right) \frac{dV_0}{d\xi} \right] R - \nu(q_1 + \nu q_2) V_0^2 + \\ & + (\nu q_1 + (3\nu - 2)q_2) \left[2\xi V_0 \frac{dV_0}{d\xi} - \frac{2\gamma}{\gamma - 1} \left(\frac{P_0}{R_0} - \frac{\xi}{R_0} \frac{dP_0}{d\xi} + \xi \frac{P_0}{R_0^2} \frac{dR_0}{d\xi} \right) \right] = 0. \end{aligned}$$

First we solve the system of homogeneous differential equations.

Fig. 1

Fig. 1 and Fig. 2

Figure 1: Fig. 1 and Fig. 2

Fig. 2

We denote these solutions by $\bar{V}, \bar{P}, \bar{R}$. From the independent variable ξ we pass to the independent variable V_0 and seek solutions in the form

$$\begin{aligned}\bar{V} &= (1 - V_0)^s \sum_{n=0}^{\infty} a_{ni} (1 - V_0)^n, & \bar{P} &= (1 - V_0)^s \sum_{n=0}^{\infty} b_{ni} (1 - V_0)^n, \\ \bar{R} &= (1 - V_0)^s \sum_{n=0}^{\infty} c_{ni} (1 - V_0)^n, & i &= 1, 2, 3, 4.\end{aligned}\quad (5)$$

We also represent ξ, P_0, R_0 as series in powers of $(1 - V_0)$

$$\begin{aligned}\xi &= \xi_n \left[1 - \frac{2}{\nu} (1 - V_0) + \dots \right], & P_0 &= P_{0n} \left[1 - \frac{2}{\nu} (1 - V_0) + \dots \right], \\ R_0 &= R_{0n} \left[1 - \frac{\nu - 1}{2\nu} \frac{R_{0n}}{\gamma P_{0n}} (1 - V_0)^2 + \dots \right];\end{aligned}\quad (6)$$

ξ_n, P_{0n}, R_{0n} are the values of the functions at the piston.

The characteristic equation of the system is

$$s^2 \left[\frac{\gamma}{\gamma - 1} \frac{P_{0n}}{R_{0n}} \left(s + \frac{1}{\nu} \right) - \frac{1}{\nu} \right] = 0. \quad (7)$$

The roots $s_1 = s_2 = 0$ correspond to solutions with a logarithmic singularity⁽²⁾. For $\gamma = 1.4$, the characteristic root $s_3 = 13/35, 5/14, 29/105$, respectively for $\nu = 1, 2, 3$.

Particular solutions of the system of differential equations with the right-hand side are likewise sought in the form of series in powers of $(1 - V_0)$.

The required solutions must satisfy the boundary conditions at the piston and at the shock wave. At the piston $u = U$, and since $V_0 = 1$, then

$V(1) = 0$. Ahead of the shock wave the gas is at rest, the density and pressure are constant, and therefore the conditions at the shock wave can be written in the form

$$\rho_1 c = \rho_2 (c - u_2), \quad p_{rr_1} = \rho_1 c u_2 + p_{rr_2}, \quad (8)$$

Fig. 3

Figure 2: Fig. 3

$$\frac{cp_1}{\gamma - 1} = \rho_1 c \left(\frac{u_2^2}{2} + \frac{p_2}{(\gamma - 1)\rho_2} \right) + p_{rr_2} u_2 + \frac{\kappa}{C_v(\gamma - 1)} \frac{\partial}{\partial r} \frac{p_2}{\rho_2},$$

where c is the velocity of the shock wave. The conditions at the shock wave after passing to dimensionless variables and varying with respect to η have the form

$$\begin{aligned} & \left(R_2 \frac{d\xi}{dV_0} \Big|_{V_0=V_0^*} - 2\xi^* a \frac{dR_0}{dV_0} \Big|_{V_0=V_0^*} \right) (1 - V_0^*) - \\ & - R_0 \left[(aV_0^* + V_2) \frac{d\xi}{dV_0} \Big|_{V_0=V_0^*} + 2a\xi^* \right] = 0 \\ & (aP_0^* + P_2) \frac{d\xi}{dV_0} \Big|_{V_0=V_0^*} - 2a\xi^* \frac{dP_0}{dV_0} \Big|_{V_0=V_0^*} - (\nu q_2 + q_1) V_0^* \frac{d\xi}{dV_0} \Big|_{V_0=V_0^*} + \\ & + (a + q_1 + q_2) 2\xi^* = V_2 \frac{d\xi}{dV_0} \Big|_{V_0=V_0^*}, \end{aligned} \quad (9)$$

$$\begin{aligned} & 2 \frac{1}{\gamma - 1} \left(\frac{P_0^*}{R_0} \frac{d\xi}{dV_0} \Big|_{V_0=V_0^*} - \xi^* \frac{d}{dV_0} \frac{P_0}{R_0} \Big|_{V_0=V_0^*} \right) + aV_0^* \left(V_0^* \Big|_{V_0=V_0^*} + 2\xi^* \right) = \\ & = \left(V_0^* V_2 - \frac{P_2}{R_0} + \frac{P_0^*}{R_0^2} R_2 \right) \frac{d\xi}{dV_0} \Big|_{V_0=V_0^*}, \end{aligned}$$

where V_0^* is the value of V_0 at the shock wave, and a is a constant.

The radius vector of the shock wave is represented in the form

$$r_2 = r_{20}(1 + a\eta + \dots), \quad (10)$$

where r_{20} is the radius vector of the shock wave for self-similar motion.

The solutions for V , P , and R have the following form, if one retains only $1 - V_0$ to the first degree:

Fig. 3

$$\begin{aligned}
 V &= (1 - V_0) \left\{ [1 + \ln(1 - V_0)] a_{11} + \left. \frac{\partial a_{11}}{\partial s} \right|_{s=0} + a_1 \right\} + \dots, \\
 P &= [1 + \ln(1 - V_0)] [b_{01} + b_{11}(1 - V_0)] + \left(\left. \frac{\partial b_{11}}{\partial s} \right|_{s=0} + b_1 \right) (1 - V_0) + \dots, \\
 R &= [1 + \ln(1 - V_0)] [c_{01} + c_{11}(1 - V_0)] + \left. \frac{\partial c_{01}}{\partial s} \right|_{s=0} + \left. \frac{\partial c_{11}}{\partial s} \right|_{s=0} (1 - V_0) + \\
 &\quad + c_{03}(1 - V_0)^{s_3} + c_0 + c_1(1 - V_0) + \dots \quad (11)
 \end{aligned}$$

In these formulas the condition at the piston has already been taken into account. The constants b_{01} , c_{03} , and a are found from the conditions at the shock wave, while the constants a_1 , b_1 , c_0 , c_1 are known coefficients of particular solutions of the inhomogeneous system.

Computations of V , P , and R were carried out for Prandtl number $\text{Pr} = 1$ for $\nu = 1, 2, 3$; they are presented in the form of graphs (see Figs. 1, 2, and 3).

Moscow Institute
of Railway Transport Engineers

Received
5 X 1965

References

1. L. I. Sedov, *Similarity and Dimensional Methods in Mechanics*, Moscow, 1961.
2. G. T. Padé, *Integration of Differential Equations*, Moscow-Leningrad, 1933.

Note: Figure translations are in progress. See original paper for figures.

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