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A. I. Akhiezer, M. P. Rekalo

1966

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Abstract

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UDC 539.12

PHYSICS

Academician of the Academy of Sciences of the Ukrainian SSR A. I. AKHIEZER,
M. P. REKALO

PHOTOPRODUCTION OF MESONS ON NUCLEONS AND $SU(6)$ SYMMETRY

In the present note, within the framework of $SU(6)$ symmetry, we investigate relations between the amplitudes for the production of mesons in a p -state in the interaction of magnetic-dipole γ -quanta with nucleons. In the case under consideration the photoproduction process is described by two partial amplitudes: one amplitude corresponds to the total angular momentum $I = \frac{1}{2}$ (M_{11}), the other to the total momentum $I = \frac{3}{2}$ (M_{13}). The spin structure of the amplitude for photoproduction of pseudoscalar mesons in the center-of-mass system, in terms of M_{11} and M_{13} , has the form ⁽¹⁾

$$F_M = i(\vec{\sigma} \cdot \vec{e} \hat{\mathbf{k}} \cdot \hat{\mathbf{q}} - \vec{\sigma} \cdot \hat{\mathbf{k}} \vec{e} \cdot \hat{\mathbf{q}})(M_{13} - M_{11}) - \vec{e} \times \hat{\mathbf{k}} \cdot \hat{\mathbf{q}}(2M_{13} + M_{11}), \quad (1)$$

where $\vec{e}$ is the polarization vector of the γ -quantum; $\hat{\mathbf{k}}, \hat{\mathbf{q}}$ are unit vectors in the directions of the momenta of the γ -quantum and of the produced meson.

In $SU(6)$ symmetry a magnetic-dipole γ -quantum is described by a tensor of second rank ⁽²⁾

$$Q_{A'}^A \equiv Q_{i'a'}^{ia} = (\vec{\sigma} \cdot \vec{e} \times \hat{\mathbf{k}})_i^a Q_{a'}^a, \quad Q_{a'}^a = \begin{pmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (2)$$

Since the mesons are produced in a p -state, it is necessary to take into account the angular spurion ⁽³⁾

$$L_{A'}^A \equiv L_{i'a'}^{ia} = (\vec{\sigma} \cdot \mathbf{q})_i^a \delta_{a'}^a, \quad (3)$$

where the Latin indices take the values 1, 2, and the Greek indices 1, 2, 3.

Then the number of independent parameters describing the process of meson photoproduction on nucleons upon absorption of a magnetic-dipole γ -quantum is determined by the number of common multiplets in the products 56×56 and $35 \times 35 \times 35$. Since

$$56 \times 56 = 1 + 35 + 405 + 2695,$$

from the product $35 \times 35 \times 35$ it is necessary to retain multiplets of dimensions 1, 35, 405, 2695. For this purpose we note that

$$35 \times 35 \times 35 = (1 + 35_1 + 35_2 + 189 + 280 + 280^* + 405) \times 35.$$

Each of the products 189×35 , 280×35 , $280^* \times 35$, 405×35 contains the multiplet 35 once. Further, the products 280×35 , $280^* \times 35$ contain the multiplet 405 once, and the product 405×35 contains the multiplet 405 twice. Thus, in the product $35 \times 35 \times 35$ there are contained 2 singlet multiplets, 9 multiplets 35, 6 multiplets 405, and one multiplet 2695. Therefore the photoproduction amplitude under consideration is determined by 18 parameters:

$$\begin{aligned} F_M = & \bar{\psi}^{A'B'C'} \{ \alpha \delta_A^A \delta_{B'}^B (LQ)_D^D M_{C'}^C + \beta M_{A'}^A Q_{B'}^B L_{C'}^C \\ & + a_1 \delta_A^A \delta_{B'}^B \delta_{C'}^C (LQ_d)_D^D M_D^{D'} + a_2 \delta_{B'}^B \delta_{C'}^C [(LQ_d)_D^A M_{A'}^D + (LQ_d)_A^D M_D^A] \\ & + a_3 \delta_{B'}^B \delta_{C'}^C [(LQ_d)_D^A M_{A'}^D - (LQ_d)_A^D M_D^A] + a_4 \delta_{C'}^C (LQ_d)_A^A M_{B'}^B + \\ & + b_1 \delta_A^A \delta_{B'}^B \delta_{C'}^C (LQ_f)_D^D M_D^{D'} + b_2 \delta_{B'}^B \delta_{C'}^C [(LQ_f)_D^A M_{A'}^D + (LQ_f)_A^D M_D^A] + \\ & + b_3 \delta_{B'}^B \delta_{C'}^C [(LQ_f)_D^A M_{A'}^D - (LQ_f)_A^D M_D^A] + b_4 \delta_{C'}^C (LQ_f)_A^A M_{B'}^B + \\ & + \delta_{B'}^B \delta_{C'}^C (c_1 R_{[A'D']}^{[AD]} M_D^{D'} + c_2 R_{[A'D']}^{AD} M_D^{D'} + c_3 R_{A'D'}^{[AD]} M_D^{D'} + c_4 R_{A'D'}^{AD} M_D^{D'}) + \\ & + d_1 \delta_{C'}^C (R_{[A'D]}^{AB} M_{B'}^D + R_{[B'D]}^{AB} M_{A'}^D) + d_2 \delta_{C'}^C (R_{A'B'}^{[AD]} M_D^B + R_{A'B'}^{[BD]} M_D^A) + \\ & + d_3 \delta_{C'}^C (R_{A'D}^{AB} M_{B'}^D + R_{B'D}^{AB} M_{A'}^D) + d_4 \delta_{C'}^C (R_{A'B}^{AD} M_D^B + R_{A'B}^{BD} M_D^A) \} \psi_{ABC}, \end{aligned} \quad (4)$$

where ψ_{ABC} is the tensor describing baryons, $M_{A'}^A$ is the tensor describing mesons,

$$(LQ_d)_{A'}^A = L_B^A Q_{A'}^B + L_{A'}^B Q_B^A, \quad (LQ)_A^A = L_B^A Q_A^B,$$

$$(LQ_f)_{A'}^A = L_B^A Q_{A'}^B - L_{A'}^B Q_B^A,$$

$$R_{[A'B']}^{[AB]} = L_{A'}^A Q_{B'}^B + L_{B'}^B Q_{A'}^A - L_{A'}^B Q_{B'}^A - L_{B'}^A Q_{A'}^B,$$

$$R_{A'B'}^{[AB]} = L_{A'}^A Q_{B'}^B - L_{B'}^B Q_{A'}^A - L_{A'}^B Q_{B'}^A + L_{B'}^A Q_{A'}^B,$$

$$R_{[A'B']}^{AB} = L_{A'}^A Q_{B'}^B - L_{B'}^B Q_{A'}^A + L_{A'}^B Q_{B'}^A - L_{B'}^A Q_{A'}^B,$$

$$R_{A'B'}^{AB} = L_{A'}^A Q_{B'}^B + L_{B'}^B Q_{A'}^A + L_{A'}^B Q_{B'}^A + L_{B'}^A Q_{A'}^B.$$

In (4) the invariant proportional to α arises from a term of the type $1(LQ) \times 35(M)$; the invariant proportional to β corresponds to the extraction of the multiplet 2695 from the product of three multiplets 35. Further, the invariants a_i and b_i ($i = 1, 2, 3, 4$) arise as a result of multiplying $35_1(LQ) \times 35(M)$ and $35_2(LQ) \times 35(M)$; the invariants c_i correspond to the extraction of the multiplet 35 from the products

$$189(LQ) \times 35, \quad 280(LQ) \times 35, \quad 280^*(LQ) \times 35, \quad 405(LQ) \times 35.$$

and, finally, d_i correspond to the extraction of the multiplet 405 from the products

$$280(LQ) \times 35, \quad 280^*(LQ) \times 35, \quad 405(LQ) \times 35.$$

The symbol (LQ) after the dimension of a multiplet means that the multiplet under consideration arises in the multiplication of the corresponding multiplets 35.

We note that from the explicit form of the tensors L and Q it follows that

$$(LQ)_A^A = 0.$$

In addition, the invariant in (4) proportional to b_1 gives no contribution to the photoproduction of pseudoscalar mesons. Thus, the amplitude for meson production in a p -state upon absorption of a magnetic-dipole γ -quantum is determined by 16 parameters, with 12 of them, as follows from (4), contributing to the combination of amplitudes $f_1 = M_{13} - M_{11}$, and 13 parameters contributing to the combination $f_2 = 2M_{13} + M_{11}$. After examination of the invariants (4), it turns out that in fact the amplitudes f_1 are determined by only 5 parameters. Thus, for the photoproduction amplitudes on the proton we have:

$$\begin{aligned}
f_1(\gamma p \rightarrow p\eta) &= 4\sqrt{6}x_1, \\
f_1(\gamma p \rightarrow p\pi^0) &= 10\sqrt{2}x_1 + 2\sqrt{2}x_3, \\
f_1(\gamma p \rightarrow n\pi^+) &= 5x_1 + 5x_2 + 4x_3, \\
f_1(\gamma p \rightarrow \Lambda K^+) &= -\frac{9}{\sqrt{6}}x_1 - \frac{9}{\sqrt{6}}x_2 - \sqrt{6}x_3 + \sqrt{6}x_5, \\
f_1(\gamma p \rightarrow \Sigma^0 K^+) &= \frac{1}{\sqrt{2}}x_1 + \frac{1}{\sqrt{2}}x_2 + \sqrt{2}x_3 + 3\sqrt{2}x_5, \\
f_1(\gamma p \rightarrow \Sigma^+ K^0) &= -2x_1 + 2x_3;
\end{aligned} \tag{5a}$$

for the photoproduction amplitudes on the neutron we have:

$$\begin{aligned}
f_1(\gamma n \rightarrow n\eta) &= -\sqrt{6}x_1 + 2\sqrt{6}x_3, \\
f_1(\gamma n \rightarrow n\pi^0) &= 5\sqrt{2}x_1 - 2\sqrt{2}x_3, \\
f_1(\gamma n \rightarrow p\pi^-) &= 5x_1 - 5x_2 + 4x_3, \\
f_1(\gamma n \rightarrow \Lambda K^0) &= 3\sqrt{6}x_1 - \frac{15}{\sqrt{6}}x_3 - \frac{1}{\sqrt{6}}x_4 + \sqrt{6}x_5, \\
f_1(\gamma n \rightarrow \Sigma^0 K^0) &= \sqrt{2}x_1 + \frac{1}{\sqrt{2}}x_3 - \frac{1}{\sqrt{2}}x_4 + 3\sqrt{2}x_5, \\
f_1(\gamma n \rightarrow \Sigma^- K^+) &= x_1 + x_2 + x_3 + x_4,
\end{aligned} \tag{5b}$$

where

$$\begin{aligned}
x_1 &= 18(b_1 + c_2 - c_3), & x_2 &= 6(b_2 + c_1 - c_4), & x_3 &= b_4, \\
x_4 &= \frac{1}{9}a, & x_5 &= \frac{1}{3}(d_1 + d_2 - d_3 - d_4).
\end{aligned}$$

From (5a) and (5b) the following relations between photoproduction amplitudes follow:

$$f_1(\gamma p \rightarrow n\pi^+) + f_1(\gamma n \rightarrow p\pi^-) = \sqrt{2}[f_1(\gamma p \rightarrow p\pi^0) - f_1(\gamma n \rightarrow n\pi^0)], \tag{6a}$$

$$f_1(\gamma p \rightarrow \Sigma^+ K^0) + f_1(\gamma n \rightarrow \Sigma^- K^+) = \sqrt{2}[f_1(\gamma p \rightarrow \Sigma^0 K^+) - f_1(\gamma n \rightarrow \Sigma^0 K^0)],$$

$$\begin{aligned}
\sqrt{2}f_1\left(\gamma p \rightarrow p \frac{\pi^0 - \sqrt{3}\eta}{2}\right) &= f_1(\gamma p \rightarrow \Sigma^+ K^0), \\
\sqrt{2}f_1\left(\gamma p \rightarrow \frac{\Sigma^0 - \sqrt{3}\Lambda}{2} K^+\right) &= f_1(\gamma p \rightarrow n\pi^+),
\end{aligned} \tag{6b}$$

$$\begin{aligned}
f_1\left(\gamma n \rightarrow n \frac{\pi^0 - \sqrt{3}\eta}{2}\right) &= -f_1\left(\gamma n \rightarrow \frac{\Sigma^0 - \sqrt{3}\Lambda}{2} K^0\right), \\
f_1(\gamma p \rightarrow p\eta) - f_1(\gamma n \rightarrow n\eta) &= \sqrt{3}f_1(\gamma n \rightarrow n\pi^0),
\end{aligned} \tag{6c}$$

$$f_1(\gamma p \rightarrow p\pi^0) + f_1(\gamma n \rightarrow n\pi^0) = \frac{\sqrt{75}}{4} f_1(\gamma p \rightarrow p\eta).$$

Let us note that the relations (6a) are a consequence of the known transformation properties of the Hamiltonian of electromagnetic interactions with respect to isotopic rotations; the relations (6b) are satisfied in $SU(3)$ -symmetry⁽⁴⁾; and, finally, the relations (6c) are valid only in $SU(6)$ -symmetry.

For an experimental test, the last relation in (6c) is of special interest. In the impulse approximation, the expression $f_1(\gamma p \rightarrow p\pi^0) + f_1(\gamma n \rightarrow n\pi^0)$ is the amplitude for photoproduction of π^0 -mesons on the deuteron. Therefore this relation between amplitudes is equivalent to a relation between the cross sections of π^0 -photoproduction on the deuteron and η -meson production on the proton, if absorption of magnetic dipole γ -quanta takes place. Such a situation occurs in the region of the first resonance of pion photoproduction on nucleons.

As for the combination of amplitudes $f_2 = 2M_{13} - M_{11}$, the relations (6a) and (6b) are satisfied for them. No new relations, valid only in $SU(6)$ -symmetry, arise.

Physical-Technical Institute
Academy of Sciences of the Ukrainian SSR

Received
6 IX 1965

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