

MOTION OF A NEUTRAL FERMION POSSESSING AN ANOMALOUS MAGNETIC MOMENT IN AN ELECTRIC FIELD

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Abstract

Full Text

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PHYSICS

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MOTION OF A NEUTRAL FERMION POSSESSING AN ANOMALOUS MAGNETIC MOMENT IN AN ELECTRIC FIELD

(Presented by Academician N. N. Bogolyubov, 18 X 1965)

Let us consider the motion of a neutral Dirac particle with anomalous magnetic moment $\mu = -\mu_0$ ($\mu_0 > 0$) in a static electric field having a component along the z -axis of the Cartesian coordinate system, $\mathcal{E}(z)$. A solution of the Dirac equation (see ¹)

$$i\hbar \frac{\partial}{\partial t} \Psi = \hat{\mathcal{H}} \Psi = \{c(\hat{\alpha} \mathbf{p}) + \rho_3 mc^2 + \mu_0 \rho_2 \sigma_3 \mathcal{E}\} \Psi \quad (1)$$

will be an eigenfunction of the Hamiltonian operator $\hat{\mathcal{H}}$, as well as of the operators $\hat{p}_x$ and $\hat{p}_y$ that commute with it:

$$\hat{\mathcal{H}} \Psi = E \Psi; \quad \hat{p}_x \Psi = \hbar k_1 \Psi; \quad \hat{p}_y \Psi = \hbar k_2 \Psi. \quad (2)$$

To characterize the spin states it is expedient to introduce the polarization-tensor operator $\hat{\Lambda} = \rho_3 [\hat{\sigma} \mathbf{p}]_3$, which also commutes with the Hamiltonian and characterizes the projection of the spin onto the direction perpendicular to the external field and to the velocity:

$$\hat{\Lambda} \Psi = \hbar \lambda \zeta \Psi \quad (3)$$

(for the transformation properties of this operator, see ²). Taking (1), (2), and (3) into account, the wave function may be represented in the form

$$\Psi = e^{-icKt} \frac{e^{i(k_1 x + k_2 y)}}{\sqrt{L_1 L_2}} \begin{pmatrix} f_1(z) \\ i\zeta e^{i\varphi} f_1(z) \\ i f_2(z) \\ \zeta e^{i\varphi} f_2(z) \end{pmatrix}, \quad (4)$$

where $c\hbar K = E$ is the particle energy; $k_1 = k \cos \varphi$; $k_2 = k \sin \varphi$; $\lambda = k = \sqrt{k_1^2 + k_2^2}$; $\zeta = \pm 1$ corresponds to the two possible spin orientations. In this case the functions f_1 and f_2 must be determined from the system of equations

$$\left\{ \frac{d}{dz} \pm \left(\frac{\mu_0}{e\hbar} \mathcal{E}(z) + \zeta K \right) \right\} f_{1,2} \mp (K - k_0) f_{2,1} = 0, \quad (5)$$

in which the upper and lower signs refer to the first and second component, respectively.

The fundamental possibility of the interaction of a neutral fermion with a static electric field of Coulomb type was noted in ³ (see also ⁴). We consider here the case when the electric field $\mathcal{E}(z)$ has the form

$$\frac{\mu_0}{e\hbar} \mathcal{E}(z) = \gamma z + \gamma_0 \quad (\gamma > 0). \quad (6)$$

Then the system of equations (5) admits the exact solution

$$f_1 = \frac{1}{2} \sqrt{1 + \frac{k_0}{K}} U_n(t), \quad f_2 = \frac{1}{2} \sqrt{1 - \frac{k_0}{K}} U_{n+1}(t), \quad (7)$$

in which $U_n(t) = \sqrt[4]{\gamma/\pi} (2^n n!)^{-1/2} e^{-t^2/2} H_n(t)$ are Hermite functions, while the variable t is equal to

$$t = \sqrt{\gamma} z + (\xi k + \gamma_0)/\sqrt{\gamma}. \quad (8)$$

In this case the energy spectrum is completely discrete and depends only on a single quantum number $n = 0, 1, 2, \dots$

$$K = \sqrt{k_0^2 + 2\gamma(n+1)}. \quad (9)$$

The solutions obtained physically correspond to the motion of the particle along a circle lying in a plane parallel to the z -axis and passing through the vector $\mathbf{p} = (p_1, p_2)$. The radius of this circle is

$$R^2 = 2 \overline{(z - \bar{z})^2} = \frac{2}{\gamma} \left(n + 1 - \frac{k_0}{2K} \right) = \frac{1}{\gamma} \left(\frac{K^2 - k_0^2}{\gamma} - \frac{k_0}{K} \right), \quad (10)$$

and the position of the center of the orbit is characterized by the quantity

$$z_0 = \bar{z} = -(\xi k + \gamma_0)/\gamma. \quad (11)$$

For definite values of the momentum $k = \sqrt{k_1^2 + k_2^2}$, the spin projection ξ , and the quantity γ_0 ($\gamma_0 = -\xi k$), the center of the circle is at the origin of coordinates.

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Note: Figure translations are in progress. See original paper for figures.

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