

# ON THE INTEGRATION OF THE EQUATIONS OF PLANE FLOW OF IDEALLY PLASTIC BODIES

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**Abstract**

**Full Text**

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*THEORY OF ELASTICITY*

**B. A. DRUYANOV**

## ON THE INTEGRATION OF THE EQUATIONS OF PLANE FLOW OF IDEALLY PLASTIC BODIES

*(Presented by Academician Yu. N. Rabotnov, June 24, 1965)*

In plastic regions, where both families of characteristics are curvilinear, the following functions of the curvilinear coordinates  $\alpha, \beta$  are to be determined:  $u, v$ —the projections of the velocity on the directions of the characteristics;  $R, S$ —the radii of curvature of the characteristics; the quantities  $\bar{x}, \bar{y}$ , related to the Cartesian coordinates  $x, y$  by the formulas  $\bar{x} = x \cos \varphi + y \sin \varphi$ ,  $\bar{y} = -x \sin \varphi + y \cos \varphi$  ( $\varphi$  is the angle of inclination of the characteristics of the family  $\alpha$  to the axis of abscissas,  $\varphi = \alpha + \beta$ ). The functions  $u, v, R, S, \bar{x}, \bar{y}$  satisfy the equation  $\partial^2 f / \partial \alpha \partial \beta + f = 0$ , for which the Riemann function is the Bessel function of the first kind of zero order

$$J_0 \left[ 2\sqrt{(\alpha - a)(\beta - b)} \right] \quad (1)$$

( $a, b$  are parameters).

The application of analytical methods, however, is complicated by the necessity of solving a chain of boundary-value problems, since usually the plastic region has a “patchwork” form. A successive solution of boundary-value problems leads to lengthy calculations. The method presented below by way of an example considerably simplifies the matter.

Consider the problem of the initial flow of a strip under indentation by a convex smooth punch. Let the punch move translationally. The field of characteristics, generalizing the known fields <sup>(2)</sup>, is shown in Fig. 1. In the region  $A_{34}A_{04}A_{15}A_{25}$  the characteristics of the family  $\beta$  are rectilinear; in the region  $A_{43}A_{40}A_{51}A_{52}$  the characteristics  $\alpha$  are rectilinear. In the remaining plastic regions the characteristics are curvilinear. The velocity distribution can be found after the actual construction of the net of characteristics in the physical plane <sup>(2)</sup>. The impermeability condition makes it possible to determine the distributed pressures along the contact arc  $A_{52}A_{25}$  and the forming straight characteristics  $A_{04}A_{15}$  and  $A_{40}A_{51}$ . The opening angle of the sector  $A_{24}A_{25}A_{15}A_{14}$ , as well as of the

Fig. 1

Figure 1: Fig. 1

sector  $A_{42}A_{52}A_{51}A_{41}$ , is determined only roughly. Therefore one may assume that along the lines  $A_{40}A_{43}$ ,  $A_{43}A_{34}$ ,  $A_{34}A_{04}$  the quantities  $\bar{x}, \bar{y}$  and their derivatives are known as functions of the coordinates  $\alpha, \beta$ . This makes it possible to determine  $\bar{x}, \bar{y}$ , and consequently the net of characteristics in the whole plastic region.

### Fig. 1

Let us determine  $\bar{x}, \bar{y}$  at the point  $M$  with coordinates  $\alpha = a$ ,  $\beta = b$ . For this it is necessary successively to determine  $\bar{x}, \bar{y}$  in the regions  $A_{43}A_{33}A_{34}$ ,  $A_{33}A_{34}A_{04}A_{03}$ ,  $A_{33}A_{43}A_{40}A_{30}$ ,  $A_{33}A_{03}A_{00}A_{30}$ . Another possibility consists in the following. Let

$$dU_x = \left( G \frac{\partial \bar{x}}{\partial \alpha} - \bar{x} \frac{\partial G}{\partial \alpha} \right) d\alpha + \left( \bar{x} \frac{\partial G}{\partial \beta} - G \frac{\partial \bar{x}}{\partial \beta} \right) d\beta. \quad (1)$$

Integrating (1) along  $A_{33}A_{34}A_{43}A_{33}$ ,  $A_{33}A_{43}ABA_{33}$ ,  $A_{33}CDA_{34}A_{33}$ ,  $A_{33}BMCA_{33}$ , we shall have

$$\begin{aligned} \int_{A_{43}A_{33}} dU_{\bar{x}} + \int_{A_{33}A_{34}} dU_{\bar{x}} + \int_{A_{34}A_{43}} dU_{\bar{x}} &= 0, \\ \int_{A_{33}A_{43}} dU_{\bar{x}} + \int_{A_{43}N} dU_{\bar{x}} + \int_{NK} dU_{\bar{x}} + \int_{KA_{33}} dU_{\bar{x}} &= 0 \end{aligned} \quad (2)$$

and so on.

Adding these equalities, we obtain an equality containing the integral of  $dU_{\bar{x}}$  along  $MDA_{34}A_{43}AM$  and integrals along the internal boundaries of these plastic regions, containing jumps of the functions  $\bar{x}$ ,  $\partial \bar{x} / \partial \alpha$ ,  $\partial \bar{x} / \partial \beta$ .

Since  $x, y$  are continuous,  $\bar{x}, \bar{y}$  are also continuous. In passing across the boundary of two regions, only the following quantities can undergo a discontinuity:  $\partial \bar{x} / \partial \alpha$  when crossing  $\beta$ , and  $\partial \bar{y} / \partial \beta$  when crossing  $\alpha$ . However, from (1) it is seen that  $dU_{\bar{x}}$  does not contain  $\partial \bar{x} / \partial \alpha$  on the lines  $\beta$ . Similarly,  $dU_{\bar{y}}$  does not contain  $\partial \bar{y} / \partial \beta$  on the lines  $\alpha$ . Consequently, after adding the equalities (2) we obtain

$$\int_{MD} dU_{\bar{x}} + \int_{DA_{34}} dU_{\bar{x}} + \int_{A_{34}A_{43}} dU_{\bar{x}} + \int_{A_{43}A} dU_{\bar{x}} + \int_{AM} dU_{\bar{x}} = 0. \quad (3)$$

Hence

$$\bar{x}_M = \frac{1}{2} (\bar{x}_A + \bar{x}_D) + \frac{1}{2} \int_{DA_{34}A_{43}A} dU_{\bar{x}}. \quad (4)$$

An analogous formula is obtained for  $\bar{y}_M$ .

For  $a = b = 0$ , at the point  $A_{00}$ ,  $\bar{x} = x, \bar{y} = y$ . Then

$$\eta_{00} = \frac{1}{\sqrt{2}}(y_{00} - x_{00}) = \frac{1}{2\sqrt{2}} (\bar{y}_{40} - \bar{x}_{40} + \bar{y}_{04} - \bar{x}_{04}) + \frac{1}{2\sqrt{2}} \int_{A_{04}A_{34}A_{43}A_{40}} dU_{(\bar{y}-\bar{x})/\sqrt{2}}.$$

Since it must be that  $\eta_{00} = 0$ , we obtain an equation imposing a condition on the parameters of the problem, for example, on the position of the points  $A_{43}, A_{34}$ .

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## REFERENCES

<sup>1</sup> R. Hill, *Mathematical Theory of Plasticity*, 1956. <sup>2</sup> B. Druyanov, *Journal of Applied Mechanics and Technical Physics*, No. 6 (1961).

*Note: Figure translations are in progress. See original paper for figures.*

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