

Integral Representations

A. A. Temlyakov

1960

SovietRxiv

View the original and related papers at <https://sovietrxiv.org/items/ru-196001.66315>

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.

Abstract

Full Text

Reports of the Academy of Sciences of the USSR
1960. Volume 131, No. 2

MATHEMATICS

A. A. TEMLYAKOV

INTEGRAL REPRESENTATIONS

(Presented by Academician M. A. Lavrent'ev on 21 XI 1959)

In preceding papers ⁽¹⁻³⁾ I constructed the theory of integral representations for functions analytic in such bicircular domains D at every boundary point of which the Levi determinant $L(\Phi) > 0$, where the boundary is given by

$$|z| - \varphi(|w|) \equiv \Phi(w, \bar{w}, z, \bar{z}) = 0,$$

i.e., for functions regular in bicircular domains whose boundaries are nowhere an analytic hypersurface. In the general case, as is known, $\varphi(r_1)$ may be constant on an interval beginning with the value $r_1 = 0$ (r_1 and $r_2 = \varphi(r_1)$ are conjugate radii of convergence) ⁽⁴⁾, and therefore this portion of the boundary of the domain D may be an analytic hypersurface. Considering the boundary of the domain D as given in the form $|w| = \psi(|z|)$, we conclude that a portion of the boundary of the domain D beginning with the value $|z| = r_2 = 0$ may also be an analytic hypersurface. Thus, in the more general case, on the portions $0 \leq |w| \leq r_1$, $|z| = R_2$, and $|w| = R_1$, $0 \leq |z| \leq r_2$, one may have $L(\Phi) = 0$, and only outside them $L(\Phi) > 0$ (we leave aside the case in which $L(\Phi)$ vanishes at isolated points of the boundary of the domain D).

However, the integral representations of both kinds ⁽³⁾ remain valid for this general case if $r_1(\tau)$ and $r_2(\tau)$ are defined in the following way. The function $r_1(\tau)$, positive and continuously differentiable on the segment $0 \leq \tau \leq 1$, satisfies the conditions: $r_1(0) > 0$; in the interval $0 < \tau < 1$, $r_1'(\tau) > 0$, and $r_1'(1) = 0$. The function $r_2(\tau)$ is defined in terms of $r_1(\tau)$ in the same way as before:

$$r_2(\tau) = R_2 \exp \left[- \int_0^\tau \frac{\tau}{1-\tau} d \ln r_1'(\tau) \right], \quad (1)$$

where R_2 is a positive constant.

Indeed, since $r_1'(1) = 0$, it may happen that $r_2(1) > 0$ ⁽¹⁾. For example,

$$r_1(\tau) = 2 - (1 - \tau)^2.$$

Then

$$r_2(\tau) = [2 - (1 - \tau)^2] \left(\frac{\sqrt{2} + (1 - \tau)}{\sqrt{2} - (1 - \tau)} \right)^{1/\sqrt{2}} (\sqrt{2} - 1)^{\sqrt{2}},$$

and we have $r_1(0) = 1$, $r_1'(1) = 0$, $r_2(1) = 2(\sqrt{2} - 1)^{\sqrt{2}}$. Taking into account that the monomials $w^m z^n$, $m \geq 0$, $n \geq 0$, are invariants with respect to the transformation

$$\frac{1}{2\pi} \int_0^{2\pi} dt \int_0^1 \frac{d}{du} [(u(r_1)u)^m (r_2(\tau)v)^n] d\tau = w^m z^n$$

for arbitrary continuous functions $r_1(\tau)$, $r_2(\tau)$ ⁽⁵⁾, and $r_2(\tau)$ is defined in the same way as before, we arrive at the conclusion that in this general case as well, taking into account $r_1(0) > 0$, $r_2(1) > 0$, the entire preceding theory of integral representations is preserved.

Thus, the integral representations

$$F(w, z) = \frac{1}{4\pi^2 i} \int_0^{2\pi} dt \int_0^1 d\tau \int_{|\zeta|=1} \frac{\Phi[r_1(\tau)\zeta^n, r_2(\tau)\eta^n]}{\zeta - u} d\zeta, \quad (2)$$

$$F(w, z) = \frac{1}{4\pi^2 i} \int_0^{2\pi} dt \int_0^1 d\tau \int_{|\zeta|=1} \frac{\zeta F[r_1(\tau)\zeta^n, r_2(\tau)\eta^n]}{(\zeta - u)^2} d\zeta, \quad (3)$$

where

$$u = \tau \left(\frac{w}{r_1(\tau)} \right)^{1/n} + (1 - \tau) \left(\frac{z}{r_2(\tau)} \right)^{1/n} e^{it},$$

$$\Phi(w, z) = F(w, z) + nwF'_w(w, z) + nzF'_z(w, z),$$

n is the least integer not smaller than

$$\sup_{0 < \tau < 1} \frac{d \ln r_1(\tau)}{d \ln \tau},$$

hold in the domain

$$D: |w| < r_1(\tau), |z| < r_2(\tau), 0 \leq \tau \leq 1,$$

i.e., in the domain bounded by three hypersurfaces:

$$0 \leq |w| \leq r_1(0), |z| = r_2(0); \quad |w| = r_1(\tau), |z| = r_2(\tau), 0 < \tau < 1; \quad |w| = r_1(1), 0 \leq |z| \leq r_2(1).$$

Here, as we see, there appears a characteristic feature of analytic functions of two complex variables: the values of the function $F(w, z)$ in the domain D are determined by its behavior, or by $\Phi(w, z)$, only on that part of the boundary of the domain $\bar{D}$ which is a nonanalytic hypersurface.

Moscow Regional Pedagogical Institute
named after N. K. Krupskaya

Received
12 XI 1959

CITED LITERATURE

- ¹ A. A. Temlyakov, *Uch. zap. Moskovsk. obl. ped. inst.*, **21**, 7 (1954).
- ² A. A. Temlyakov, *Izv. AN SSSR, Ser. Mat.*, No. 21, 89 (1957).
- ³ A. A. Temlyakov, *DAN*, **120**, No. 5 (1958).
- ⁴ B. A. Fuks, *Theory of Analytic Functions of Several Complex Variables*, Moscow, 1948.
- ⁵ A. A. Temlyakov, *DAN*, **129**, No. 5 (1959).

Note: Figure translations are in progress. See original paper for figures.

Source: Math-Net.Ru and CyberLeninka. Machine translation. Verify with the original.