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Abstract

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PHYSICS

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ON A POSSIBLE DIPOLE MOMENT OF TRANSITION IN Λ -PARTICLES

At the present time it may apparently be regarded as experimentally proven ⁽¹⁾ that the system of two π -mesons arising in the decay of the θ^+ -meson has spin 0 and positive parity, whereas the system of 3 π -mesons arising in the decay of the τ^+ -meson is in the state 0^- . This compels one to think that (if parity nonconservation is not assumed) θ and τ are different particles. To explain the existence of two K -mesons differing in parity and having very close masses, the suggestion ⁽²⁾ was made that, in addition to two kinds of K -mesons (θ and τ), there also exist two kinds of hyperons (Λ_θ and Λ_τ , Σ_θ and Σ_τ), with all interactions (except weak ones) invariant under the simultaneous replacement $\theta \rightarrow \tau$ and $\Lambda_\theta \rightarrow \Lambda_\tau$, $\Sigma_\theta \rightarrow \Sigma_\tau$.

If such an assumption is adopted, then the mass difference between Λ_θ and Λ_τ , caused only by weak interactions, will prove to be extremely small (of order 10^{-6} eV), so that the possibility arises of transitions $\Lambda_\theta \rightleftharpoons \Lambda_\tau$ under the action of an external electromagnetic field*.

In the case where the electromagnetic field changes little over the extent of the Λ -particle, transitions of this kind effectively reduce to the appearance of a dipole moment of transition. It is clear that such a dipole moment $\mathbf{d}$ can be directed only along the spin $\mathbf{s}$ of the Λ -particle, so that

$$\mathbf{d} = er_0 \mathbf{s} C_p, \quad (1)$$

where r_0 is some constant with the dimension of length, determined by the dimensions of the Λ -particle (everywhere below $\hbar = c = 1$), and C_p is the operator of permutation of Λ_θ and Λ_τ ⁽²⁾: $C_p = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

The presence of a dipole moment in the Λ -particle may lead to a number of observable phenomena.

Let us first consider the scattering of Λ -particles by the Coulomb field of a nucleus. The interaction energy has the form

$$U = -\frac{Ze^2r_0}{r^3} \mathbf{rs}C_p. \quad (2)$$

Restricting ourselves to the Born approximation** and assuming the Λ -particle to be nonrelativistic, we find for the scattering amplitude the expression

* We proceed here from the most general assumption that each of the mesons θ and τ interacts with both kinds of hyperons, so that, for example, both θ and Λ_θ , and θ and Λ_τ , may be produced together.

** The estimate of the applicability of the Born approximation is obtained from the usual condition $|U| \ll v/r$. Taking as r the minimum distance—the nuclear radius R —we find from (2) the upper estimate:

$$E \gg \frac{1}{8}(Ze^2)^2 \left(\frac{r_0}{R}\right)^2 m.$$

In practically interesting cases this condition is satisfied fairly well: thus, for $Z = 30$ and $r_0 = 1/\mu_\pi$, it reduces to $E \gg 0.5$ MeV.

$$f_{\text{Coul}} = 2iZe^2r_0m \frac{\mathbf{sq}}{q^2}, \quad (3)$$

where m is the mass of the Λ -particle; q is the momentum transferred to the nucleus, $q = 2p \sin(\theta/2)$. If the spin of the Λ -particle is taken to be $1/2$, the differential cross section is

$$d\sigma_{\text{Coul}} = \frac{1}{4}\pi(Ze^2)^2r_0^2 \frac{m}{E \sin^2(\theta/2)} \sin \theta d\theta. \quad (4)$$

The main role in the total cross section is played by the region of small angles. Therefore the total cross section can be obtained by integrating (4) over the interval of angles corresponding to a change of the impact parameter from the nuclear radius R to the Bohr radius a . This gives

$$\sigma_{\text{Coul}} = \pi(Ze^2)^2r_0^2 \frac{m}{E} \ln \frac{a}{R}. \quad (5)$$

A numerical estimate shows that (for $r_0 \sim 1/\mu_\pi = 1.4 \cdot 10^{-13}$ cm) the cross section for Coulomb scattering with the transition $\Lambda_\theta \rightarrow \Lambda_\tau$, at Λ -particle energies of the order of tens of megaelectronvolts, exceeds the nuclear one. Thus, for $Z \sim 30$ and $E \sim 10$ MeV, $\sigma_{\text{Coul}}/\sigma_{\text{nucl}} \sim 5$. If Λ_θ and Λ_τ have substantially

different lifetimes, then the presence of a strong Coulomb interaction could be observed by placing a thin plate of material with large Z in the path of long-lived Λ -particles and observing an increase in the number of Λ -decays.

It is interesting to note that, because of the dependence of the Coulomb-scattering amplitude (3) on the spin and because of the interference of Coulomb scattering with nuclear scattering, polarization of Λ -particles can arise in the plane perpendicular to the incident beam. The degree of polarization is then determined by the equality

$$\mathbf{P} = -\frac{\mathbf{q}}{q^2} \frac{2Ze^2r_0m \operatorname{Im} f_0}{|f_0|^2 + (Ze^2r_0m/q)^2}, \quad (6)$$

where f_0 is the amplitude of nuclear scattering with the transition $\Lambda_0 \rightarrow \Lambda_\tau$. In addition, in the case when the spin of the Λ -particle is greater than $1/2$, the presence in (3) of the term $s\mathbf{q}$ will lead to the appearance of an “aligned” beam of Λ -particles, i.e., for example in the case $s = 3/2$, after scattering the number of Λ -particles with spin projection on $\mathbf{q}$ equal to $\pm 3/2$ will be greater than the number of Λ -particles with spin projection equal to $\pm 1/2$.

Another phenomenon to which the presence of a transition dipole moment in the Λ -particle can lead is the appearance of levels in the Coulomb field of the nucleus. Indeed, since the potential energy $U \sim Ze^2r_0/r^2$, and the kinetic energy $\sim 1/mr^2$, then for $Ze^2r_0m \gtrsim 1$, one may expect levels to arise.

In this case the Schrödinger equation outside the nucleus has the form

$$\left\{ -\frac{1}{2m}\Delta - \frac{Ze^2r_0}{2r^3} \bar{\mathbf{r}}\sigma C_p \right\} \psi = E\psi. \quad (7)$$

Instead of the two-component functions $\psi = \begin{pmatrix} \psi_{\Lambda_\theta} \\ \psi_{\Lambda_\tau} \end{pmatrix}$, it is convenient to introduce the functions

$$\varphi = \frac{1}{2}(\psi_{\Lambda_\theta} - \psi_{\Lambda_\tau}) \quad \text{and} \quad \chi = \frac{1}{2}(\psi_{\Lambda_\theta} + \psi_{\Lambda_\tau}),$$

in which equation (7) assumes the diagonal form

$$\left\{ -\frac{1}{2m}\Delta + \frac{Ze^2r_0}{2r^3} \mathbf{r}\sigma \right\} \varphi = E\varphi \quad (8)$$

and an analogous equation for χ . We shall confine ourselves to consideration of the system of S -levels. We shall seek the solution in the form

$$\varphi = \frac{1}{r} \left[u(r) + \frac{\vec{\sigma}\vec{r}}{r} w(r) \right]. \quad (9)$$

Substitution of (9) into (8) leads to the following system of equations for the wave functions outside the nucleus:

$$d^2u/dr^2 - \alpha w/r^2 = \kappa^2 u, \quad (10a)$$

$$d^2w/dr^2 - 2w/r^2 - \alpha u/r^2 = \kappa^2 w, \quad (10b)$$

where $\alpha = Ze^2r_0m$ and $\kappa^2 = 2m|E|$. Equations (10a, b) can be solved by putting $w = Cu$, where C is a constant. The values of the constant C are determined in such a way that (10a) coincides with (10b). This can be achieved if C satisfies the equation

$$\alpha C = \alpha/C + 2, \quad (11)$$

whence

$$C_{1,2} = (1 \mp \sqrt{1 + \alpha^2})/\alpha. \quad (12)$$

The general solution is then written in the form $u = u_1 + u_2$, $w = C_1 u_1 + C_2 u_2$, where u_1 and u_2 obey the equations

$$u'_{1,2} - \alpha C_{1,2} u_{1,2}/r^2 = \kappa^2 u_{1,2}. \quad (13)$$

The solutions of equations (13) that vanish as $r \rightarrow \infty$ can be written in the following form ³:

$$u_1 = A\{J_{iq}(i\kappa r) - e^{-\pi q} J_{-iq}(i\kappa r)\}; \quad q = \sqrt{(\alpha^2 + 1)^{1/2} - 5/4}, \quad (14a)$$

$$u_2 = B\{e^{i\pi p} J_{-p}(i\kappa r) - J_p(i\kappa r)\}; \quad p = \sqrt{(\alpha^2 + 1)^{1/2} + 5/4}. \quad (14b)$$

We shall assume that inside the nucleus the interaction is described by a potential well of depth V . The solutions of the corresponding equations for the region inside the nucleus will then be

$$u = A_1 \sin kr, \quad w = B_1 [\sin kr/kr - \cos kr], \quad k^2 = 2m(V - |E|). \quad (15)$$

Solutions (14) and (15) must be matched at the boundary of the nucleus R . We shall consider such levels for which $\kappa R \ll 1$, since only for them is the Λ -particle outside the nucleus and the level energy determined mainly by the Coulomb forces. Since we are interested in the case of heavy nuclei, one may

take $kR \gg 1$. Using these two conditions and matching solutions (14) and (15) at $r = R$, we arrive at the following equation for determining the eigenvalues κ :

$$kR = -\frac{1}{2} \frac{\alpha^2}{\sqrt{1+\alpha^2}(1+\sqrt{1+\alpha^2})} \left\{ \left[\operatorname{ctg} kR - \frac{(1+\sqrt{1+\alpha^2})^2}{\alpha^2} \operatorname{tg} kR \right] \times \right. \\ \left. \times \left[\frac{1}{2} + q \operatorname{ctg} \left(q \ln \frac{\kappa R}{2} - \psi \right) \right] - \left(\frac{1}{2} - p \right) \left(-\frac{(1+\sqrt{1+\alpha^2})^2}{\alpha^2} \operatorname{ctg} kR + \operatorname{tg} kR \right) \right\}; \\ \psi = \frac{1}{2i} \ln \frac{\Gamma(iq+1)}{\Gamma(-iq+1)}. \quad (16)$$

On the left-hand side of (16) stands the large quantity kR . This requires that $\operatorname{ctg} (q \ln \frac{\kappa R}{2} - \psi)$, which appears on the right-hand side, be large. Hence, for the energy

for the n -th level we find

$$E_n = \frac{2}{mR^2} \exp \left\{ -2 \frac{n\pi - \psi}{\sqrt{(\alpha^2 + 1)^{1/2} - 5/4}} \right\}. \quad (17)$$

As follows from (17), bound states arise when $(\alpha^2 + 1)^{1/2} > 5/4$, i.e., when $Ze^2 r_0 m > 3/4$. Formula (17) is valid for $n \gg 1$. If one assumes $r_0 \sim 1/\mu_\pi$, then for a heavy nucleus ($Z \sim 80$) the energy of the first level will be of the order of 50–100 keV. It is not difficult to calculate the total number of levels. The last levels must correspond to the motion of a Λ -particle at distances of the order of the Bohr radius of the atom a , i.e., $\sqrt{2mE_n} a \sim 1$. Thus, the total number of levels is determined by the relation

$$\frac{n\pi - \psi}{\sqrt{(\alpha^2 + 1)^{1/2} - 5/4}} \sim \ln \frac{a}{R} \quad (18)$$

and is (for $r_0 \sim 1/\mu_\pi$) about 7 for heavy nuclei.

Let us note that the broadening of levels associated with the absorption of Λ -particles by nuclei when $(kR)^{-1} \ll 1$ turns out to be small, since, as follows from (16), in this case it is immaterial whether k is real or complex.

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CITED LITERATURE

¹ J. Orear, G. Harris, S. Taylor, Phys. Rev., **102**, 1676 (1956). ² T. D. Lee, C. N. Yang, Phys. Rev., **102**, 290 (1956). ³ M. Morse, H. Feshbach, *Methods of Theoretical Physics*, part II, 1953, p. 1665.

Note: Figure translations are in progress. See original paper for figures.

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